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Charge Dynamics in Junction Arrays

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abstract: 2D Junction arrays with nearest neighbour capacitance larger than the self capacitance are a physical realization of the 2D Coulomb gas. One therefore expects a Kosterlitz-Thouless-Berezinskii (KTB) transition, where charge dipoles dissociate. We performed simulations of the charge dynamics in such arrays, based on tunnel rates for single electrons and Cooper pairs. The resulting I-V curves clearly show the Coulomb gap at low temperatures, while at higher temperatures one finds effects of the KTB transition.

2D arrays of Josephson junctions have long been studied as a realization of the XY-model, which can be mapped onto a 2D Coulomb gas (CG) of vortices. Nowadays it is possible to fabricate arrays in which the charging energy E_C exceeds the Josephson coupling E_J [1]. The electrostatic energy E_{elst} of a configuration of charges Q_i on the ith island is: $E_{elst} = \frac{1}{2}\sum_{i,j} Q_i \left(C^{-1}\right)_{i,j} Q_j$. If the nearest neighbour capacitance C is much larger than the self capacitance C_0, the inverse capacitance matrix $C^{-1}_{i,j}$ behaves logarithmically up to a scale set by the screening length $\Lambda = \sqrt{C/C_0}$. Therefore the electric charges in these arrays provide a *direct* physical realization of a 2D CG.

The dynamics, incoherent tunneling of charges between neighbouring islands i and j, is governed by tunnel rates. For quasi-particles (QP) and Cooper pairs (CP) they are [2]:

$$\Gamma^{ij}_{QP} = (\Delta E_{ij}/e^2\hbar R_t)\ \left[\exp\left(\Delta E_{ij}/k_b T\right) - 1\right]^{-1}$$

$$\Gamma^{ij}_{CP} = \frac{\pi E_J^2}{4\alpha_s \hbar \Delta E_{ij}} \frac{1-\exp\left(-\Delta E_{ij}/k_b T\right)}{\left(\pi/\alpha_s\right)^2 + \sinh^2(\frac{1}{2}\Delta E_{ij}/k_b T)}\ , \text{ for } \alpha_s \gg 1\ , \tag{1}$$

where $\alpha_s = R_q/R_s$ measures the effective shunt resistance R_s seen by the junction ($\alpha_s \gg 1$ for a generic array), $\alpha_t = R_q/R_t$ measures the tunnel resistance R_t and $R_q = \frac{h}{4e^2}$ is the quantum resistance. ΔE_{ij} is the difference in energy of the *whole* array before and after a tunnel event (global rule). Tunneling of a charge in the direction of the voltage drop will generally cost energy if the external voltage is lower than the Coulomb gap voltage $V_g \sim N_s \frac{e}{C}$ (N_s is the number of junctions in series)[3]. At low temperatures the rates (1) vanish for $\Delta E_{ij} > 0$ and for $V \leq V_g$ no current will flow, i.e. Coulomb blockade. At higher voltages the rate Γ_{QP} leads to diffusive motion for QP; the rate Γ_{CP} is peaked at $\Delta E_{ij} = 0$, reflecting the resonant nature of CP tunneling for $\alpha_s \gg 1$.

As the system is a 2D CG, there is a charge unbinding transition for QP at $k_b T_c = \frac{E_C}{4\pi\epsilon}$ ($k_b T_c = \frac{E_C}{\pi\epsilon}$ for CP), where ϵ is a dielectric constant slightly larger than one and $E_C = e^2/2C$. For $T < T_c$ the conductance is nonlinear for small voltages, i.e. $I \sim V^{a(T)}$ [4]. At the transition the exponent a jumps from 3 to 1. For $T > T_c$ conductance is linear, i.e. $I \sim V$, and proportional to the density n of free (unpaired) charges, which follows a square root cusp relation: $n \sim \exp\{-b(T/T_c - 1)^{-\frac{1}{2}}\}$; b is a constant of order unity.

Real time simulations based on the rates (1) yield I-V curves for QP and CP as shown in figs. 1a and 2. For low temperatures a Coulomb gap indeed develops; fig. 2 shows the resonant structure for CP. The presence of a fixed thermal distribution of QP on the islands weakens the Coulomb gap for CP (fig. 2b). The density of charges n follows the theoretical prediction (fig.1b) if the applied voltage is low. Since a finite voltage $V_c = \frac{e}{\pi C}$ is needed to separate pairs with the largest separation (the system size), there is only a restricted range of voltages where the jump in the

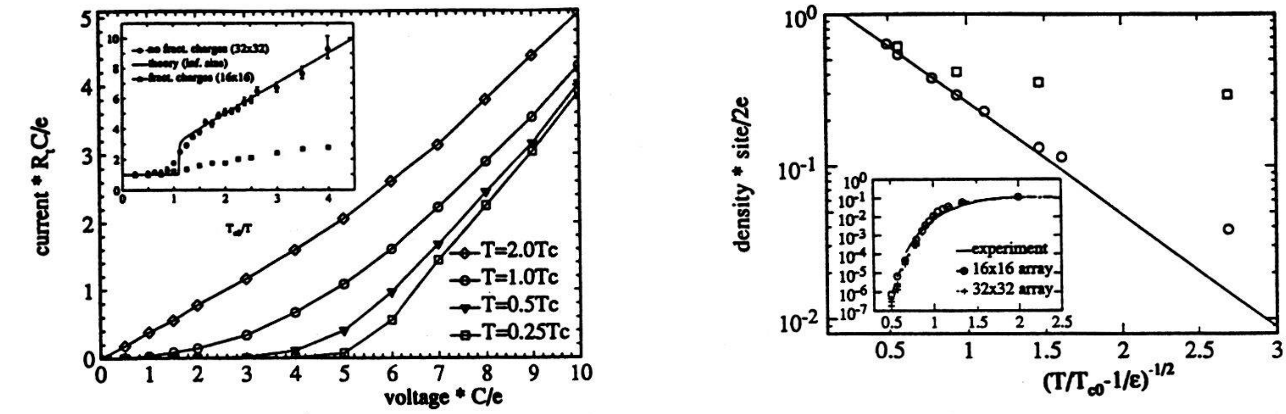

Figure 1: **a:** I-V curves for QP in a 32x32 array, the inset shows the exponent a in $I \sim V^a$ vs. inverse temperature. **b:** Charge density for $T > T_c$ vs. temperature for CP in 16x16 array, α_s =25, 'boxes': same with fixed QP background; the inset shows conductance x R_t vs. temperature $/\epsilon T_c$ for CP.

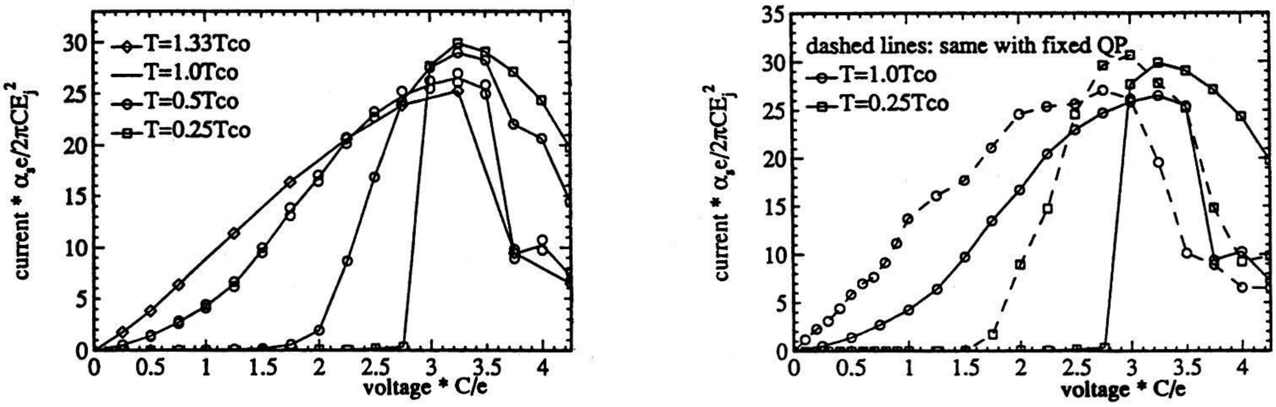

Figure 2: **a:** IV curves for CP in a 16x16 array, α_s =10. **b:** same, dashed lines denote fixed QP background.

exponent of the nonlinear conductance can be observed. The exponent a is shown as a function of temperature in the inset of fig. 1a. The correspondence to the theoretical prediction is good for QP in a 32x32 system. Random fractional offset-charges wash out the transition.

In order to make closer contact to experiment, it would be interesting to obtain IV-curves for both CP and QP tunneling at the same time. If the ratio of the tunnel rates for QP and CP is varied, one expects a transition as a function of R_t or T from QP dominated to CP dominated behaviour [5].

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