Is the Kosterlitz-Thouless Transition 'Exotic'?

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Abstract. Theoretical methods used in the $2DXY$ Kosterlitz-Thouless (KT) transition, applied to the anisotropic-coupling $3DXY$ model, yield scaling equations for elliptical vortex loops, with a $1/R$ segment-segment interaction. The transition temperature, calculated in terms of the (critically irrelevant) coupling anisotropy, is in good agreement with previous Monte Carlo work. The KT transition is thus seen as part of a larger class of topological excitation dominated phase transitions. Possible application include layered superconductors, and (2+1)D quantum Josephson arrays.

Introduction

The KT transition in 2D systems is dominated by ±1 vortex unbinding, and is hence regarded as unconventional, or 'exotic'. However Monte Carlo work by Shrock [1] and Dasgupta [2] and co-workers indicates that topological excitations (vortex loops and spin hedgehogs) play a central role in even <u>three</u> dimensional XY and Heisenberg models. Simulations of the 3D XY model show vortex loop excitations do exist, grow, and proliferate at $T = T_c$. Epiney finds that [3] the transition temperature in the anisotropic 3D case, $T_c(\delta_o)$, decreases smoothly to the 2D KT value, as the anisotropy parameter $\delta_o = 1 - K_\perp/K_{||} \to 1$.

Layered 3D XY Model

The anisotropic 3D XY model is $(\beta = 1/k_BT)\beta H = -\sum_\mu \sum_i K_\mu(\cos\Delta_\mu\theta_i - 1)$ where Δ_μ is a discrete derivative in the $\mu = x, y, z$ direction and i a site on a 3D cubic lattice of lattice constant a_o. The coupling constants describe layers, $\vec{K} = (K_{||}, K_{||}, K_\perp)$. An anisotropy parameter is $\delta_o = 1 - K_\perp/K_{||}$, with $\delta_o = 0$ for the isotropic case and $\delta_o = 1$ for decoupled layers. At the spin wave level, $\beta H \approx \frac{1}{2}\sum_{\mu,\vec{q}}(q_\mu a_\mu)^2|\theta_{\vec{q}}|^2$, with $\vec{a} = (a_o, a_o, a_1)$ where $a_1 \equiv a_o/(1-\delta_o)^{1/2}$.

Applying the standard dual transform of Savit [4] to the model one obtains, on dual cubic lattice, [5]

$$\beta H \approx \frac{\pi}{2}K_{||}(1-\delta_o)\sum_\mu \sum_{r_{||},r'_{||}} \sum_{z,z'} J_\mu(\vec{r}_{||}, z)J_\mu(\vec{r}'_{||}, z')U(\vec{r}_{||} - \vec{r}'_{||}, z - z')$$
$$+ \frac{\pi}{2}K_{||}\delta_o \sum_{r_{||},r'_{||}} \sum_{z,z'} J_z(r_{||}, z)J_z(r'_{||}, z')U(r_{||} - r'_{||}, z - z') \tag{1}$$

Here $\vec{J}(\vec{r}_{||}, z) = \pm1$ are segments of closed vortex loops. The interaction is $U(\vec{r}_{||}, z) = [(\vec{r}_{||}/a_1)^2 + (z/a_o)^2]^{-1/2}$. For $\delta_o = 0$ one recovers previous [6,7] loop results. For $\delta_o \to 1$, one has $U(\vec{r}_{||}, z) \to -2(lnr_{||})\delta_{z,o} + 0((r_{||}/a_1)^2 lnr_{||})$, and one recovers the 2D KT interaction, with 'charges' $m(r_{||}, z) \equiv J_z(r_{||}, z) = 0, \pm1$ in decoupled, overall-neutral planes. The $\vec{J}_{||}$ segments cost zero free energy in this limit.

The self-energy of elliptical loops with minor axis at an angle α to the z axis, from Equ.(1), gives a bare loop fugacity, (B=5.631) that peaks at $\alpha = \frac{\pi}{2}$, in agreement with Friedel [8],

$$y_o(\alpha) \approx exp\{-BK_o(1-\delta_o)^{1/2}[1 + \frac{1}{2}\frac{\delta_o}{(1-\delta_o)}\sin^2(\alpha - \frac{\pi}{2})]\} \tag{2}$$

In Equ.(2) the isotropic part coefficient scales just as in the $\delta_o = 0$ case isotropic coupling K_l [6,7], with however the initial value $K_{l=0} = K_{||}(1-\delta_o)^{1/2}$. The anisotropic part is irrelevant, and critical exponents [7] are unchanged, for any nonzero $1-\delta_o$. With increasing anisotropy, the fugacity of Equ.(2) increases, and the transition temperature is pushed down. The bare critical coupling $K_{||}(T_c) = E_{||}/k_B T_c$, using isotropic case results [7] is ($K_{oo} = 0.454, A = 4.197, B = 5.631$),

$$K_{||}(1-\delta_o)^{1/2} = K_{oo} + A[e^{-BK_{||}(1-\delta_o)^{1/2}} e^{\eta} I_o(\eta) - e^{-BK_{oo}}] \tag{3}$$

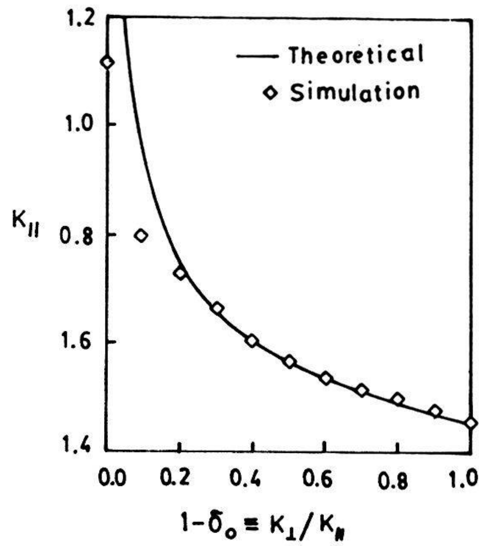

Fig. 1. *Inverse transition temperature $K_{||} = E_{||}/k_B T_c$ versus inter-plane coupling $(1-\delta_o) = K_\perp/K_{||} = E_\perp/E_{||}$.*

Here $\eta \equiv B\delta_o K_{||}/4(1-\delta_o)^{1/2}$ and $I_o(\eta)$ is a zeroth-order imaginary-argument Bessel function, arising from an angular average of Equ.(2). Equ.(3) does not include Hikami-Tsuneto [9] type single-plane rectangular excitations, that contribute for $(1-\delta_o) \ll 0.2$ Fig.1 shows a plot of the critical $K_{||}$ versus $1-\delta_o$, (solid line) compared with some of the MC data of Epiney (diamonds). Agreement is also good for $1 < K_\perp/K_{||} < 15$, (not shown). There are no fitted parameters.

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REFERENCES

1. G. Kohring, R. Shrock, and P. Wills, Phys. Rev. Lett. **57**, 1538 (1987).

2. M.-H. Lau and C. Dasgupta, Phys. Rev. B**39**, 7212 (1989).

3. J.Epiney, Preprint (1990) "3D XY Model Near Criticality," IPS 90-08, ETH Zurich.

4. R. Savit Phys. Rev. B**17**, 1340 (1978).

5. This is also the zero charge limit of the lattice superconductor, S.E. Korshunov, Europhys. Lett. **11**, 757 (1990).

6. G. Williams, Phys. Rev. Lett. **59**, 1926 (1987).

7. S.R. Shenoy, Phys. Rev. B **40**, 5056, (1989); Phys. Rev. B**42**, 8595 (1990).

8. J. Friedel, J. Phys. (Paris) **49**, 1561 (1988); **49** 1769 (1988).

9. S. Hikami and T. Tsuneto, Prog. Theor. Phys. **63**, 387 (1980).