Minimax Credibility

By A. Marazzi, Swiss Federal Institute of Technology, Zürich

This paper is a summary of the last three chapters of my doctoral dissertation [1]. In the first two chapters explicit formulae are derived (using multidimensional techniques as in [2]) for the coefficients of a credibility estimator of the form

$$a \times (\text{total claims}) + b \times (\text{number of claims}) + c \times (\text{average claim size}) + d.$$

This estimator is used to forecast total losses in the future and is based upon past observations. The asymptotic behaviour (when the number of years tends to infinity) is also examined. Numerical computations show that there is no appreciable difference in the Bayes risk when the simpler formulae with $c = 0$ or $c = 0$ and $b = 0$ are used. Only these simpler formulae are therefore considered in the rest of the dissertation. A method is suggested here to compute optimal (minimax) credibility estimates when the structure function is not exactly known: one merely assumes that it belongs to a given set of distributions. An introductory report on these ideas has alrady been given by Prof. *H. Bühlmann* in [3]. We develop here a more general theory and we describe a numerical method to compute the minimax solution.

1. Model, Notations and Credibility Formulae

First we give a short summary of the assumptions and formulae presented in [2] and we introduce some notations. We consider the risk performance of a risk associated with a parameter (η, θ) during one year. The generalization to n years is straightforward.

Random variables	*Distributions given* (η, θ)
Number of claims K	$P(k \mid \eta) = \text{Prob}(K = k \mid \eta)$
Claim sizes $Y^1 \dots Y^K$	$S(y \mid \theta) = \text{Prob}(Y^h < y \mid \theta) \quad h = 1 \dots K$
Total claims $X = \sum_{h=1}^{K} Y^h$	$G(x \mid \eta, \theta) = \text{Prob}(X < x \mid \eta, \theta)$

Hypotheses

1) η, θ are independent random variables with distribution $U(\eta, \theta) = T(\eta)\, Q(\theta)$.
2) Given (η, θ) the random variables $\{K, Y^1, Y^2 \dots\}$ are independent.
   Given θ the random variables $\{Y^1, Y^2 \dots\}$ are independent.
3) All the random variables and parameters associated with different risks are independent.

Expectations

We consider a function f (for ex. an estimator of $E(X \mid \eta, \theta)$) defined on the range Ω of the vector $(K, Y^1 \dots Y^K)$ associated with the risk (η, θ). In the following we assume that the expectations are always defined. Then

$$E(f \mid \eta, \theta) = \int_{\Omega} f(\omega)\, dP(\omega \mid \eta, \theta) = \sum_{k=0}^{\infty} P(k \mid \eta) \int f(k, y_1 \dots y_k) \prod_{h=0}^{k} dS(y_h \mid \theta),$$

where $$\int \prod_{h=0}^{0} dS(y_h \mid \theta) = 1.$$

Notations

$$E\{E(Y^h \mid \theta)\} = \int E(Y^h \mid \theta)\, dQ(\theta) = m_Q$$
$$E\{\mathrm{Var}(Y^h \mid \theta)\} = v_Q$$
$$\mathrm{Var}\{E(Y^h \mid \theta)\} = w_Q$$

Similarly we define m_T, v_T, w_T and m_U, v_U, w_U for the random variables K and X and the distributions T and U. We also use the following notations:

$$E\{E^2(Y^h \mid \theta)\} = z_Q \quad \text{and similarly} \quad z_T \quad \text{and} \quad z_U.$$

(a, d) estimate: an estimate $l(K, Y^1 \dots Y^K)$ for $E(X \mid \eta, \theta)$ of the form

$$l(K, Y^1 \dots Y^K) = aX + d \tag{1}$$

(a, b, d) estimate: an estimate $l(K, Y^1 \dots Y^K)$ for $E(K \mid \eta, \theta)$ of the form

$$l(K, Y^1 \dots Y^K) = aX + bK + d \tag{2}$$

$$\begin{aligned}
r((a, d), U) &= E\{(aX + d - E(X \mid \eta, \theta))^2\} \\
&= a^2 v_U + (1-a)^2 w_U + (d - (1-a)\, m_U)^2 \\
r((a, b, d), U) &= E\{(aX + bK + d - E(X \mid \eta, \theta))^2\} \\
&= a^2 (m_T v_Q + v_T w_Q) + (1-a)^2 z_T w_Q + ((1-a)\, m_Q - b)^2 w_T \\
&\quad + (a m_Q + b)^2 v_T + ((1-a)\, m_U - b m_T - d)^2
\end{aligned}$$

H: range of η; Z: range of θ.

Credibility Formulae

From $r((a_0, d_0), U) = \operatorname*{Min}_{(a,d)} r((a, d), U)$ we get

$$a_0 = \frac{w_U}{v_U + w_U}, \quad d_0 = (1 - a_0)\, m_U.$$

From $r((a_0, b_0, d_0), U) = \operatorname*{Min}_{(a,b,d)} r((a, b, d), U)$ we get

$$a_0 = \frac{m_T^2 w_Q + w_T w_Q}{m_T^2 w_Q + w_T w_Q + m_T v_Q + v_T w_Q}, \quad a_0 + \frac{b_0}{m_Q} = \frac{w_T}{v_T + w_T},$$

$$a_0 + \frac{b_0}{m_Q} + \frac{d_0}{m_U} = 1.$$

In order to evaluate the credibility estimates it is necessary to know the structure function $U(\eta, \theta)$. According to the empirical Bayes point of view it is possible to estimate U on the basis of actual collateral data. Instead of considering point estimates for the parameters m_U, v_U, z_U, etc. as in the existing credibility literature we merely assume that the data lead us to "sets of possible" structure functions and parameters. We will then define a statistical game where the nature chooses a structure function in these sets and the statistician chooses an estimator of the form (1) or (2). In the next section we indicate some possible definitions of sets of structure functions. We consider only sets containing the factor Q of U; the sets for T are analogously defined.

2. Estimation of the Structure Function

Case 1. Estimates for m_Q, v_Q, z_Q can be easily constructed (see [4] for a distribution-free method). Suppose that $[m_{Q_1}, m_{Q_2}]$, $[v_{Q_1}, v_{Q_2}]$, $[z_{Q_1}, z_{Q_2}]$ are the corresponding α%-confidence intervals. Finding these intervals is however an open problem. We define

$$\mathrm{P}_Q^1 = \{Q \mid m_Q \in [m_{Q_1}, m_{Q_2}],\ z_Q \in [z_{Q_1}, z_{Q_2}],\ v_Q \in [v_{Q_1}, v_{Q_2}]\}.$$

According to the Bonferroni inequality $(P(\bigcap_i A_i) \geq 1 - \sum_i P(A_i^C))$ P_Q^1 covers the true distribution Q' with probability greater than $1 - 3(1-\alpha)$ %.

Case 2. Suppose that the distribution of Y^h given θ belongs to a given parametric family $\{S(y \mid \theta) \mid \theta \in Z\}$. We consider a confidence region R (for ex. of Kolmogorov type) that covers the distribution $S(y) = \int S(y \mid \theta)\, dQ'(\theta)$ with a certain probability (Q' is the true distribution). Under the hypothesis that R can be bounded by two (possibly improper) distributions S_1 and S_2 (i.e. for ex. S_1 increasing, continuous from left, but $S_1(-\infty) \neq 0$ or $S_1(+\infty) \neq 1$):

$$\mathrm{R} = \{S'' \mid S_1(y) \geq S''(y) \geq S_2(y)\}$$

we define

$$\mathrm{P}_Q^2 = \{Q \mid \int S(y \mid \theta)\, dQ(\theta) \in \mathrm{R}\}.$$

Case 3. Without observations the pure Bayesian will guess an a-priori distribution $Q(\theta)$. In our opinion it is easier and safer to give a set of possible structure functions:

$$\mathrm{P}_Q^3 = \{Q \mid Q_1(\theta) \geq Q(\theta) \geq Q_2(\theta) \text{ for all } \theta\}$$

where Q_1 and Q_2 are distributions such that $Q_1(\theta) \geq Q_2(\theta)$ for all θ.

Case 4. Suppose that a fraction $1-\delta$ of the risk population is described by a distribution $Q_1 \in \mathrm{P}_Q^i$ $(i = 1, 2, 3)$ and the rest by a distribution $Q_2 \in \mathrm{P}_Q^j$ $(i \neq j = 1, 2, 3)$. Then the whole population is described by

$$Q \in \mathrm{P}_Q^{ij} = \{Q \mid Q = \delta Q_2 + (1-\delta) Q_1\}.$$

Case 5. We consider sets P_Q and P_T (for T) as in the prededing cases. We set

$$\mathrm{P}_U = \{U \mid U = TQ,\ T \in \mathrm{P}_T,\ Q \in \mathrm{P}_Q\}.$$

3. The Game of the Actuary Against Nature

Let us now define a game $(\mathrm{L}, \mathrm{P}_U, r)$ of the actuary against nature, where
L is the set of the estimators available to the actuary, that is:

$$\mathrm{L} = \{l = (a, d)\} \quad \text{or} \quad \mathrm{L} = \{l = (a, b, d)\},$$

P_U is the set of the possible structure functions for the nature,
r is the loss function defined by

$$r(l, U) = \begin{cases} r((a, d), U) & \text{if } \mathrm{L} = \{(a, d)\} \\ r((a, b, d), U) & \text{if } \mathrm{L} = \{(a, b, d)\}. \end{cases}$$

We assume at first that the measures $P(\omega \mid \eta, \theta)$ i.e. the families $\{S(y \mid \theta)\}$ and

$\{P(k \mid \eta)\}$ are known. It is not unreasonable for the actuary to use a minimax rule l_0 for which:

$$\operatorname*{Max}_{U \in \mathrm{P}_U} r(l_0, U) = \operatorname*{Min}_{l \in \mathrm{L}} \operatorname*{Max}_{U \in \mathrm{P}_U} r(l, U).$$

Let K_U be the closed convex hull of P_U (with the weak topology). Under assumptions which are satisfied in all the important practical cases it is shown in [1] that the game $(\mathrm{L}, \mathrm{K}_U, r)$ has a value and a minimax pair (l_0, U_0). Because the loss is convex attention may be restricted to nonrandomized Bayes rules.
The straightforward calculation of a minimax rule is in general too complicated. We shall present a method which allows to characterize a least favorable distribution U_0 in P_U. If we can show that U_0 is also least favorable in K_U then we have a minimax solution. In general this is unfortunately not possible and a computational control is necessary.

4. Calculation of a Least Favorable Structure Function

We examine only some examples related to the cases described in section 2. Consider the following situations for the variables Y^h and the distribution Q:

$(i)_y$: no relationship is known between $\operatorname{Var}(Y^h \mid \theta)$ and $E(Y^h \mid \theta)$, that is no relationship is known between v_Q and (m_Q, z_Q) ("non parametric case"). We suppose that $Q \in \mathrm{P}^1_Q$.

$(ii)_y$: Hypothesis: $\operatorname{Var}(Y^h \mid \theta) = k_0 + k_1 E(Y^h \mid \theta) + k_2 E^2(Y^h \mid \theta)$, where k_0, k_1 and k_2 are known positive constants. For many important distributions in the field of insurance this hypothesis is known to be satisfied. Q belongs to P^1_Q or to P^3_Q.

$(iii)_y$: The distribution of Y^h given θ belongs to a family $\{S(y \mid \theta)\}$ and Q belongs to P^2_Q or to P^3_Q. We still suppose that $\operatorname{Var}(Y^h \mid \theta) = k_0 + k_1 E(Y^h \mid \theta) + k_2 E^2(Y^h \mid \theta)$.

Similarly we can consider situations $(i)_k$, $(ii)_k$, $(iii)_k$ for K and T (replace above $y, Y^h, Q, \theta, \ldots$ by $k, K, T, \eta, \ldots$).
Remark. In the situations $(i)_k$, $(ii)_k$, $(i)_y$, $(ii)_y$ we do not assume that the families $\{P(k \mid \eta)\}$ and $\{S(y \mid \theta)\}$ are known (except for the constants k_0, k_1 and k_2). In our statistical game we may then interpret the strategies of the nature as triplets $(U, \{P(k \mid \eta)\}, \{S(y \mid \theta)\})$ (the nature chooses also the families $\{P(k \mid \eta)\}$ and $\{S(y \mid \theta)\}$) or as vectors $\vec{u} = (m_T, v_T, z_T, m_Q, v_Q, z_Q)$ (r depends on U only via $\vec{u}$) but we still denote them by U.
In order to guarantee the existence of a least favorable distribution we may

require (possibly by additional restrictions such as compactness of $H \times Z$) that P_U is compact.

Let $rm(U) = \underset{l \in L}{\mathrm{Min}}\, r(l, U)$. One notices that in the case

$(i)_y$ the function rm is increasing in z_Q and v_Q (m_Q fixed)
$(ii)_y$ the function rm is increasing in z_Q (m_Q fixed)
$(iii)_y$ the function rm is increasing in z_Q (m_Q fixed)
Similar results may be obtained if we consider rm as a function of m_T, v_T, z_T. In all the cases the problem of finding a maximum of rm over the set P_U can therefore be reduced to the problem of finding a maximum of a function of two variables m_T and m_Q. In the cases $(i)_y - (i)_k$ it suffices to choose $z_{Q_0} = \mathrm{Max}\, z_Q$, $v_{Q_0} = \mathrm{Max}\, v_Q$, $z_{T_0} = \mathrm{Max}\, z_T$, $v_{T_0} = \mathrm{Max}\, v_T$. In the cases $(ii)_y$ or $(iii)_y - (ii)_k$ or $(iii)_k$ it is possible to calculate the function $z_{Q_0}(m_Q) = \mathrm{Max}\, \{z_{Q'} \mid Q' \in P_Q, m_{Q'} = m_Q\}$ (resp. $z_{T_0}(m_T)$). One chooses then the pair (m_{T_0}, m_{Q_0}) which maximizes rm. The cases with $Q \in P^2_Q$ or $T \in P^2_T$ require some numerical computations based on linear parametric optimization (see for ex. [5]). At first the problem is discretized: the distribution $Q_0(\theta)$ and the function $z_{Q_0}(m_Q)$ are computed only for a finite set of arguments and the linear condition $Q \in P^2_Q$ (i.e. $\int S(y \mid \theta)\, dQ(\theta) \in R$) is imposed only on a finite set of points $\theta_1 \ldots \theta_n$ (resp. $y_1 \ldots y_{n*}$). Under suitable assumptions one can show that the solution of the discretized problem converges weakly to the solution of the true problem.
If $E(Y^h \mid \theta) = \theta$ and Z is a real closed interval the case $Q \in P^3_Q$ allows an explicit solution based on the following
Lemma. The distribution Q_m for which $z_{Q_m} = \mathrm{Max}\, \{z_Q \mid Q \in P^3_Q, m_Q = m\}$ is given by

$$Q_m(\theta) = \begin{cases} Q_1(\theta) & \text{for } \theta \leq \theta_0 \\ q & \text{for } \theta_0 < \theta \leq \theta_1 \\ Q_2(\theta) & \text{for } \theta_1 < \theta \end{cases}$$

where the parameters θ_0, q and θ_1 are determined so that $m_{Q_m} = m$.

5. On the Existence of a Saddlepoint for the Game (L, P_U, r)

If the set P_U is convex then the saddlepoint theorem holds for the game (L, P_U, r) and we obtain easily a minimax estimate. All the sets P_Q and P_T considered in section 2 are convex. Unfortunately if P_Q and P_T are convex the set $P_U = \{U \mid U = TQ,\ T \in P_T,\ Q \in P_Q\}$ is convex if and only if at least P_T

or P_Q contains exactly one element. Without this restriction it is possible to show the existence of a saddlepoint only for some special cases. We notice here merely that they include the important case $P(k \mid \eta)$ Poisson, $S(y \mid \theta)$ unknown and $l = (a, d)$.
In general we must perform an approximate numerical control as follows.

Define $$m_{Q_1} = \operatorname*{Min}_{Q \in P_Q} m_Q, \qquad m_{Q_2} = \operatorname*{Max}_{Q \in P_Q} m_Q$$

$$P_Q^* = \{Q \in P_Q \mid z_Q = z_{Q_0}(m_Q), m_{Q_1} \leq m_Q \leq m_{Q_2}, (v_Q = \operatorname*{Max}_{Q \in P_Q} v_Q \text{ in the case } (i)_y)\}.$$

Define similarly m_{T_1}, m_{T_2} and P_T^*. Let

$$\begin{aligned} P_U^* &= \{U \mid U = TQ, T \in P_T^*, Q \in P_Q^*\} \\ \overline{Q} &= [m_{T_1}, m_{T_2}] \times [m_{Q_1}, m_{Q_2}] \end{aligned}$$

$r_1(l, (m'_T, m'_Q)) = r(l, U)$ for all $U \in P_U^*$ such that $U = TQ, m_T = m'_T, m_Q = m'_Q$.
We have:

a) The game (L, P_U, r) has a saddlepoint if and only if the game (L, P_U^*, r) has a saddlepoint. If the two saddlepoints exist then they coincide.
b) The game (L, P_U^*, r) is equivalent to the game $(L, \overline{Q}, r_1)$.

Let (m_{T_0}, m_{Q_0}) be the least favorable strategy in the game $(L, \overline{Q}, r_1)$ constructed by the method of section 4 and let l_0 be the corresponding Bayes solution. If $r_1(l_0, (m_T, m_Q)) \leq r_1(l_0, (m_{T_0}, m_{Q_0}))$ holds for all $(m_T, m_Q) \in \overline{Q}$ then $(l_0, (m_{T_0}, m_{Q_0}))$ is a saddlepoint and l_0 is minimax. This inequality can be verified on a fine grid in $\overline{Q}$.

6. An Example

We compute numerically the distribution

$$Q_m \in M = \{Q \mid S_2'(y) \leq \int S(y \mid \theta)\, dQ(\theta) \leq S_1'(y), m_Q = m\}$$

for which $z_{Q_m} = \operatorname{Max} z_Q$, $m \in [m_{Q_1}, m_{Q_2}]$. We suppose that:

a) $$S(y \mid \theta) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_0^y \frac{1}{z} \exp\left(-\left(\ln z - \ln \theta + \frac{\sigma^2}{2}\right) \Big/ 2 \Big/ \sigma^2\right)^2 dz \text{ with } \sigma = 1.$$

Notice that $E(Y^h \mid \theta) = \theta$, $\operatorname{Var}(Y^h \mid \theta) = \theta^2(e-1)$ so that $v_Q = z_Q(e-1)$.

b) $Z = \{\theta_j \mid j = 1 \dots 5\} = \{1, 1.5, 2, 2.5, 3\}$. Because Z is finite we do not consider a further discretization of Z.

c) $S_2'(y) = [S(y) - 0.1]^+$, $S_1'(y) = 1 + [S(y) + 0.9]^-$,

where $S(y) = \frac{1}{5} \sum S(y|\theta_j)$ and $[.]^+$ ($[.]^-$) denote the positive (resp. negative) part.

Let $y_1 = 0.20$, $y_2 = 0.56$, $y_3 = 1$, $y_4 = 1.72$, $y_5 = 2.5$, $y_6 = 3.44$, $y_7 = 5$, $y_8 = 6.56$ be the discretization of the range of Y^h. By linear parametric optimization we obtain the solutions $(t_{m1}, t_{m2} \dots t_{m5})$ of the problems

$$\sum_{j=1}^{n-1} (\theta_j^2 - \theta_{j+1}^2)\, t_{mj} + \theta_n^2\, t_{mn} = \text{Max}\,!$$

$$\sum_{j=1}^{n-1} (\theta_j - \theta_{j+1})\, t_{mj} + \theta_n\, t_{mn} = m$$

$$\left.\begin{array}{l} S_2'(y_i) \le \sum_{j=1}^{n-1} (S(y_i \mid \theta_j) - S(y_i \mid \theta_{j+1}))\, t_{mj} + S(y_i \mid \theta_n)\, t_{mn} \le S_1'(y_i) \\ 0 \le t_{m1} \le t_{m2} \le \dots \le t_{mn} = 1 \\ i = 1 \dots n^*, \end{array}\right\} \qquad (3)$$

where $m \in [m_{Q_1}, m_{Q_2}]$. We put

$$Q_m(\theta) = \begin{cases} 0 & \text{for } \theta \le \theta_1 \\ t_{mi} & \text{for } \theta_i < \theta \le \theta_{i+1}. \\ 1 & \text{for } \theta > \theta_n \end{cases}$$

By linear optimization of the object function

$$\sum_{j=1}^{n-1} (\theta_j - \theta_{j+1})\, t_j + \theta_n\, t_n$$

under the side conditions (3) (with $t_{mj} = t_j$) we can also find m_{Q_1} and m_{Q_2}. The following table gives us the results of the parametric optimization for our example.

m	z_{Q_m}	$Q_m(1)$	$Q_m(1.5)$	$Q_m(2)$	$Q_m(2.5)$	$Q_m(3)$
1.450	2.125	0	0.099	1.000	1.000	1.000
1.664	3.658	0	0.667	0.667	0.667	0.667
2.607	7.428	0	0.196	0.196	0.196	0.196

For further values of m one can interpolate linearly.

MIN

m_Q \ m_T	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
1.4501	5.24	5.44	5.61	5.76	5.89	5.75	5.57	5.35	4.96	4.51	3.84
1.5658	9.11	9.49	9.86	10.20	10.52	10.56	10.56	10.54	10.35	10.13	9.71
1.6645	12.17	12.70	13.20	13.68	14.14	14.28	14.40	14.48	14.40	14.28	13.96
1.7972	13.76	14.35	14.91	15.45	15.97	16.11	16.22	16.30	16.17	16.00	15.60
1.9129	15.04	15.68	16.29	16.88	17.44	17.57	17.66	17.72	17.54	17.31	16.81
2.0286	16.24	16.92	17.58	18.20	18.79	18.90	18.97	18.99	18.75	18.44	17.82
2.1443	17.33	18.06	18.75	19.40	20.02	20.10	20.12	20.09	19.77	19.36	18.60
2.2530	18.32	19.08	19.80	20.48	21.12	21.15	21.12	21.02	20.59	20.06	19.13
2.3757	19.20	19.99	20.73	21.43	22.08	22.05	21.95	21.77	21.21	20.54	19.41
2.4914	19.97	20.77	21.53	22.24	22.89	22.78	22.59	22.31	21.60	20.76	19.39
2.6071	20.61	21.43	22.19	22.90	23.54	23.34	23.04	22.64	21.75	20.71	19.07

CONTROL

m_Q \ m_T	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
1.4501	14.08	13.72	13.35	12.99	12.63	12.04	11.46	10.88	10.20	9.53	8.76
1.5658	16.66	16.50	16.34	16.18	16.01	15.55	15.08	14.61	14.02	13.43	12.70
1.6645	18.86	18.87	18.88	18.89	18.90	18.54	18.17	17.80	17.28	16.75	16.07
1.7972	19.17	19.27	19.36	19.46	19.56	19.22	18.88	18.54	18.03	17.51	16.81
1.9129	19.44	19.61	19.78	19.96	20.13	19.81	19.50	19.19	18.68	18.17	17.45
2.0286	19.72	19.96	20.21	20.45	20.70	20.41	20.12	19.84	19.33	18.83	18.10
2.1443	19.99	20.31	20.63	20.95	21.26	21.01	20.75	20.49	19.99	19.49	18.75
2.2530	20.26	20.65	21.05	21.44	21.83	21.60	21.37	21.13	20.64	20.15	19.39
2.3757	20.53	21.00	21.47	21.94	22.40	22.20	21.99	21.78	21.29	20.81	20.04
2.4914	20.81	21.35	21.89	22.43	22.97	22.79	22.61	22.43	21.95	21.47	20.68
2.6071	21.08	21.69	22.31	22.93	23.54	23.39	23.23	23.08	22.60	22.13	21.33

A parametric optimization of z_T gave the following results.

m'	3.0	3.4	3.7	3.9	4.0
$z_{T_{m'}}$	12.44	15.40	16.90	17.70	18.0

We compute now the least favorable pair (m_{T_0}, m_{Q_0}) in the game $(\mathrm{L}, \overline{\mathrm{Q}}, r_1)$ with

$$\mathrm{L} = \{(a, d)\}, \quad \overline{\mathrm{Q}} = [3.0, 4.0] \times [1.450, 2.607]$$

$z_{Q_0}(m)$ defined by linear interpolation between the points (m, z_{Q_m}) of the first table

$z_{T_0}(m')$ defined by linear interpolation between the points $(m', z_{T_{m'}})$ of the second table.

The matrix MIN represents the function $\underset{(a,d)}{\mathrm{Min}}\, r_1((a, d), (m_T, m_Q))$ on an 11×11 grid in $\overline{\mathrm{Q}}$.
By further subdivisions of the grid we find a local maximum at $m_{Q_0} = 2.607$, $m_{T_0} = 3.4$ with the value 23.54. We obtain $l_0 = \left(a_0, \dfrac{d_0}{m_{T_0} m_{Q_0}}\right) = (0.34, 0.66)$.

The Matrix CONTROL represents the function $r_1(l_0, (m_T, m_Q))$ on the grid of MIN. The inequality $r_1(l_0, (m_T, m_Q)) \leq 23.54$ holds for all the points of the grid and the obvious monotonicity of the function in all directions suggest strongly that the inequality holds also on the whole domain $\overline{\mathrm{Q}}$. The point $(l_0, (m_{T_0}, m_{Q_0}))$ should therefore be a saddlepoint.

Bibliography

[1] *Marazzi, A.*: Minimax Credibility, Dissertation an der ETH Zürich, Diss. Nr. 5695 (1976).

[2] *Bühlmann, H.*: A Comparison of Three Credibility Formulae Using Multidimensional Techniques, The ASTIN Bulletin, Vol. VII (1974).

[3] – Minimax Credibility, in Credibility Theory and Applications, Academic Press (1975).

[4] *Bühlmann, H.* und *Straub, E.*: Glaubwürdigkeit für Schadensätze, Mitteilungen Vereinigung Schweizerischer Versicherungsmathematiker, Band 70 (1970).

[5] *Gass, S.I.*: Linear Programming, McGraw-Hill, New York (1958).

Alfio Marazzi
Fachgruppe für mathematische Statistik
ETH Zürich
Clausiusstr. 59
8006 Zürich

Zusammenfassung

Es wird eine Methode vorgeschlagen, um optimale (minimax) Credibility-Schätzungen zu berechnen, wenn die Strukturfunktion nicht exakt bekannt ist; es wird nur angenommen, dass sie zu einer vorgegebenen Menge von Verteilungsfunktionen gehört. Es wird auch eine numerische Methode für die Berechnung der Minimax-Lösung beschrieben.

Résumé

Une méthode est proposée pour calculer des estimateurs de Credibility optimales (minimax) lorsque la vraie distribution de structure n'est pas exactement connue. On suppose seulement qu'elle appartient à un ensemble donné de distributions. Une méthode numérique pour calculer la solution minimax est également décrite.

Riassunto

Viene proposto un metodo per calcolare stime di Credibility ottimali (minimax) qualora non si conosca esattamente la funzione di struttura ma si sappia solo che appartiene ad un dato insieme di distribuzioni. Viene anche descritto un metodo numerico per calcolare la soluzione minimax.

Summary

A method is suggested to compute optimal (minimax) Credibility estimates when the structure function is not exactly known; one merely assumes that it belongs to a given set of distributions. A numerical method to compute the minimax solution is also described.