Graduation, bootstrap and confidence intervals

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Introduction

The familiar graduation methods used in actuarial science provide us point estimates, and the methods fall short in providing confidence intervals. In this paper, we would like to supplement them with a bootstrap confidence interval and see their coverage accuracy and interval lengths, and how their shapes follow the point estimates.

In section 1, we review the four graduation methods to be used: moving weighted average minimum R_z^2, cross validation polynomial regression, supersmoother, and basis-spline.

In section 2, we give a brief introduction to the bootstrap methods. There are many versions of bootstrap methods: for instance, percentile, bias-corrected accelerated, bootstrap t and short bootstrap t methods. In this paper, the percentile method is used to produce confidence intervals, and the other methods will be applied in a sequel paper.

In section 3, we apply the graduation methods to 1979 English Life Table No. 13 (male). Although the graduation methods used here have been around for some time, this paper uses them together for the first time on a common data set. These methods can then also be compared in terms of smoothness, squares of residuals, mean weighted sum of squares of residuals and cross-validated scores.

In section 4, we outline some future work.

1 Graduation Methods

As the following methods are quite familiar to the actuaries, extensive quote of literature does not seem necessary. On each method , we will include only a few representative names.

A. *Moving-weighted-average minimum* R_z^2 *method* (see *Greville*, 1974)

The $2m+1$ term m-w-a formula is given by

$$\hat{y}_i = \sum_{j=-m}^{m} a_j y_{i+j}$$

where the coefficients are determined by the required properties that the formula reproduces polynomials of degrees up to a certain degree d, usually cubic, and minimizes the zth differences of graduated values. Usually z is taken as 2 or 3.
In essence, the graduation problem is solved by minimizing

$$\sum_{j=-n-z}^{n} (\Delta^z a_j)^2$$

subject to the condition

$$\sum_{j=-m}^{m} a_j = 1 \qquad \text{and} \qquad \sum_{j=-m}^{m} j^2 a_j = 0,$$

where $\triangle^z$ is the zth difference operator.

B. *Cross-validation regression method* (see *Stone*, 1974)

The cross validation method is used to determine the degree d of the estimated polynomial regression model. For instance, for a given d, let the polynomial regression model be

$$y = \beta_0 + \beta_1 x + \cdots + \beta_d x^d + e$$

where e is the error term, and let

$$\hat{y}(i) = \hat{\beta}_0{}^{(i)} + \hat{\beta}_1{}^{(i)} x + \cdots + \hat{\beta}_d{}^{(i)} x^d$$

be its least squares estimated polynomial based on $n-1$ data points, $(x_1, y_1), \ldots, (x_{i-1}, y_{i-1}), (x_{i+1}, y_{i+1}), \ldots, (x_n, y_n)$, i.e. all data except the ith pair are used in the estimation. The y_i is saved to cross validate the $\hat{y}_i{}^{(i)}$, the value of $\widehat{y}(i)$ at x_i. We can then obtain the weighted cross-validation score associated with the degree d as

$$\text{cvs}\ (d) = \frac{1}{n}\sum_{i=1}^{n} w_i(y_i - \widehat{y}_i^{(i)})^2 .$$

The optimal value d is the degree which gives the minimum cross-validation score among the various scores of $d = 0, 1, 2, \ldots, n-2$.

C. *Supersmoother* (see *Friedman*, 1984)

This method is a varying span moving average method, similar to the m-w-a method above, except that the span, J_i, which is the number of observations being included in averaging the y_j values around y_i, can be chosen individually. The graduation formula can be described as

$$\widehat{y}_i = \frac{1}{J_i}\sum_{j=i-(J_i/2)}^{j=i+(J_i/2)} y_j$$

where the J_i are chosen to minimize the weighted average

$$\frac{1}{n}\sum_{i=1}^{n} w_i(y_i - \widehat{y}_i)^2 .$$

D. *Basis-spline* (see *Silverman*, 1985)

This method produces a fitted curve f which minimizes

$$\frac{1}{n}\sum_{i=1}^{n} w_i(f(x_i) - y_i)^2 + k\int_a^b (f''(x))^2\, dx ,$$

where f is a sum of basis splines which are cubic polynomials on each interval (x_i, x_{i+1}); at each x_i, the first and second order derivatives f' and f'' are continuous. This is analogous to Whittaker-Henderson graduation, using

f'' rather than z-th difference as a measure of smoothness. The constant k, which balances the two goals of fit and smoothness, is determined by cross-validation.

2 Bootstrap methods

Since *Efron* (1981) introduced this Monte Carlo simulation method, bootstrap has caught the attention of both theoretical and applied statisticians. Discussion of the following methods can be found in *Efron* (1985, 1986, 1987). The latest paper by *Hall* (1988) put the various bootstrap methods in a unified framework and derived their theoretical properties. *Portnoy* (1987) applied bootstrap to investigate the cross over in sex-distinct mortality rates.

A. *Standard error estimation*

In the different graduation methods we have different estimates of the true values underlying the given data set. In order to answer the question how good are these graduated values as estimates of the true values, we may rely on bootstrap samples. Assume that the (x_i, y_i) are related by $y_i = f(x_i) + e_i$, where f is unknown, and $e_i \sim N(0, \sigma_i)$. The bootstrap algorithm proceeds as follows:

a. Use $\widehat{y}_i$, the graduated value as a point estimate of $f(x_i)$, and residue $r_i^2 = (y_i - \widehat{y}_i)^2$. We can smooth the r_i^2 over i and then use it as an estimate of the variance σ_i^2.

b. For each i, generate an independent observation e_i^* from $N(0, \sigma_i)$, and let $y_i^* = \widehat{y}_0 + e_i^*$. The bootstrap sample $(x_1, y_1^*), (x_2, y_2^*), \ldots, (x_n, y_n^*)$ is then considered as a new set of data points. The sample is smoothed to produce $\widehat{y}_1, \widehat{y}_2, \ldots, \widehat{y}_n$.

c. When step b is repeated, say, 100 times, then for each i, the mean and variance of $\widehat{y}_i$ can be approximated respectively by

$$\mathrm{Av}\,(\widehat{y}_i^{\,*}) = \frac{1}{100}\left(\sum \text{ the } 100\,\widehat{y}_i^{\,*} \text{ values }\right).$$

$$\mathrm{Var}\,(\widehat{y}_i^{\,*}) = \frac{1}{100}\left(\sum \text{ the } 100\,(\widehat{y}_i^{\,*} - \mathrm{Av}\,(\widehat{y}_i^{\,*}))^2 \text{ values }\right).$$

After smoothed over i, Av$(\hat{y}_i^*)$ and Var$(\hat{y}_i^*)$ are the bootstrap mean and variance.

B. Confidence intervals constructions

From the above estimate standard error, we have several ways to construct confidence intervals:

a. Standard percentile method
We use the 100α point of a standard normal variate. For instance, the 90 % central confidence interval is

$$\text{Av}(\hat{y}_i^*) + 1.65\sqrt{\text{var}(\hat{y}_i^*)}\ .$$

b. Percentile t method
It is similar to the above, except that the t table is used for the 100α percentile point.

c. Bootstrap t method
For each i, let TAV$(\hat{y}_i^*)$ be the 25 % trimmed mean, and Iq$(\hat{y}_i^*)$ be the distance between the 75th and 25th percentiles of $\hat{y}_i^*$. We can smooth TAV$(\hat{y}_i^*)$ and Iq$(\hat{y}_i^*)$ over i. The bootstrap t 90 % confidence interval is

$$[\text{TAV}(\hat{y}_i^*) - \text{Iq}(\hat{y}_i^*)\varrho^{.05},\ \text{TAV}(\hat{y}_i^*) + \text{Iq}(\hat{y}_i^*)\varrho^{.95}],$$

where $\varrho^{.95}$ and $\varrho^{.05}$ are estimated from the critical points in the empirical distribution of the bootstrap values.

d. Bias corrected method & bias corrected accelerated method
The bias corrected method assumes that normality and constant standard error can be achieved by some transformation, $z = g(y)$ and $\hat{z} = g(\hat{y})$. Then $(\hat{z} - z)/t \sim N(-z_0, 1)$, with t being the constant standard error of $\hat{z}$, and z_0 the bias constant, and z will have the confidence interval $\hat{z} + tz_0 + tz^{\alpha}$. The confidence interval for y is obtained by the inverse transformation $y = g^{-1}(z)$. The bias corrected accelerated method assumes there is some g which normalizes the standard error, with the result $(\hat{z} - z)/t \sim N(-z_0\sigma_t, \sigma_t)$ where $\sigma_t = 1 + at$ for some bias constant a.

3 The English Life Table Example

We now apply the above methods with The 1979 English Life Table No. 13, for males age 2–40. We set $i = x_i$, with i ranging from 2 to 40. Using crude mortality rate u_i, exposure E_i, graduated mortality rates b_i from The 1969 English Life Table No. 12, for males age 2–40, and weight w_i which is defined as $E_i/b_i(1-b_i)$, we graduate $y_i = u_i - b_i$. The end result is the graduated mortality $\widehat{u}_i$ which is obtained by $\widehat{u}_i = \widehat{y}_i + b_i$.

A. Numerical results of the graduation methods

a. Moving-weighted average minimum R_z^2
We use $z = 3$ and $m = 5$, i.e. a symmetric eleven term formula with coefficients $(a_{-5}, \dots, a_0) = (-.0279, -.0268, .0357, .1413, .2387)$. We use the natural method (Greville 1981) to extend y_i for graduating the end values.
These extended y_i are:

$$y_i = c_1 y_{i+1} + c_2 y_{i+2} + \cdots + c_5 y_{i+5}, \quad \text{for } i = 1, 0, \dots, -3, \quad \text{and}$$
$$y_i = c_1 y_{i-1} + c_2 y_{i-2} + \cdots + c_5 y_{i-5}, \quad \text{for } i = 41, 42, \dots, 45,$$

where

$$(c_1, c_2, \dots, c_5) = (1.1608,\ 0.2811,\ -0.1410,\ -0.2045,\ -0.0964).$$

b. Cross-validation polynomial regression
Computation by Brooks et al. (1986) showed that for the same set of data the degree 3 gives minimum cross-validation score. The estimated polynomial regression is $y = -0.001985 + 0.3155x - 1.7086x^2 + 0.02435x^3$.

c. Supersmoother
Computation was done using an algorithm developed by *J. Friedman* and *W. Stuetzle* of Stanford University.

d. Basis-Spline
Computation was done using a package developed by *F. Sullivan* of University of California, Berkeley, using cubic splines.

The graduated values from all these four methods are given in Table 1.

Table 1

The ungraduated values $y_i = u_i - b_i$ and graduated values $\hat{y}_i$ and $\hat{u}_i$ for the 1979 English Life Tables No. 13 (males ages $i = 2$ to 40), where the u_i are the crude mortality rates, the b_i are the graduated mortality rates for the 1969 English Life Table No. 12, and the w_i are the weights.

					GRADUATED VALUES							
					Cross-Validation		Moving-Weighted-Average		Supersmoother		Basis-Spline	
i	$10^6 y_i$	$10^6 u_i$	$10^6 b_i$	$10^{-6} w_i$	$10^6 \hat{y}_i$	$10^6 \hat{u}_i$	$10^6 \hat{y}_i$	$10^6 \hat{u}_i$	$10^6 \hat{y}_i$	$10^6 \hat{u}_i$	$10^6 \hat{y}_i$	$10^6 \hat{u}_i$
2	-187	803	990	1205.5	-144.9	845.1	-153.8	826.2	-120.5	869.5	-109.2	880.8
3	- 64	626	690	1757.7	-122.8	567.2	-120.5	569.2	-110.9	579.1	-100.5	589.5
4	-106	514	620	1989.8	-103.7	516.3	- 96.4	523.6	-101.4	518.6	- 92.0	528.0
5	-108	462	570	2201.8	- 87.4	482.6	- 81.1	488.9	- 91.1	478.9	- 83.7	486.3
6	- 53	467	520	2428.7	- 73.9	446.1	- 72.4	447.3	- 82.3	437.7	- 75.8	444.2
7	- 78	413	480	2616.8	- 62.8	417.2	- 66.7	413.3	- 73.9	406.1	- 68.6	411.4
8	- 71	369	440	2811.3	- 64.2	375.8	- 61.4	378.6	- 65.9	374.1	- 62.1	377.9
9	--50	360	410	2955.9	- 47.8	362.2	- 56.0	354.0	- 59.9	350.1	- 56.5	353.5
10	- 55	335	390	3027.5	- 43.5	346.5	- 51.9	330.0	- 55.0	335.0	- 51.8	338.2
11	- 46	334	380	3027.5	- 41.2	338.8	- 49.6	330.4	- 48.3	331.7	- 48.3	331.7
12	- 52	328	380	2955.6	- 40.7	339.3	- 48.9	331.1	- 43.2	336.8	- 46.0	334.0
13	- 36	374	410	2682.4	- 41.8	368.2	- 43.5	336.5	- 40.6	369.4	- 45.0	365.0
14	- 30	440	470	2285.6	- 44.6	425.4	- 30.8	439.2	- 39.4	430.6	- 45.6	424.4
15	- 78	512	590	1775.3	- 48.5	541.5	- 16.4	573.6	- 41.6	548.4	- 47.7	542.3
16	31	811	780	1323.9	- 53.7	726.3	- 10.7	769.3	- 47.8	732.2	- 51.5	728.5
17	46	1036	990	1033.0	- 60.1	929.9	- 19.8	970.2	- 56.3	933.7	- 56.8	933.2
18	- 86	1034	1120	906.9	- 67.4	1052.6	- 45.1	1074.9	- 68.6	1051.4	- 63.5	1056.5
19	- 91	1079	1170	865.1	- 75.4	1094.6	- 77.6	1092.4	- 82.0	1088.0	- 71.3	1098.7
20	-122	1068	1190	861.2	- 84.1	1105.9	-110.3	1079.7	- 97.0	1093.0	- 79.7	1110.3

Table 1 continued

					GRADUATED VALUES							
					Cross-Validation		Moving-Weighted-Average		Supersmoother		Basis-Spline	
i	$10^6 y_i$	$10^6 u_i$	$10^6 b_i$	$10^{-6} w_i$	$10^6 \hat{y}_i$	$10^6 \hat{u}_i$	$10^6 \hat{y}_i$	$10^6 \hat{u}_i$	$10^6 \tilde{y}_i$	$10^6 \tilde{u}_i$	$10^6 \tilde{y}_i$	$10^6 \tilde{u}_i$
21	-119	1061	1180	894.6	- 93.3	1086.6	-136.3	1043.7	-108.9	1071.1	- 88.5	1091.5
22	-151	989	1140	973.0	-102.9	1037.1	-148.8	991.2	-119.7	1020.3	- 97.4	1042.6
23	-127	953	1080	1082.6	-112.6	967.4	-148.2	931.8	-126.7	953.3	-106.2	973.8
24	-202	818	1020	1131.6	-122.4	897.6	-137.3	882.7	-131.1	888.9	-144.6	875.4
25	-120	870	990	1111.3	-132.1	857.9	-122.7	867.3	-133.6	856.4	-122.8	867.2
26	- 67	913	980	1045.8	-141.6	838.4	-115.8	864.2	-139.6	840.4	-130.7	849.3
27	- 84	916	1000	998.2	-150.7	849.3	-120.6	879.4	-145.4	854.6	-138.5	861.5
28	-172	868	1040	918.3	-159.3	880.7	-134.9	905.1	-150.7	889.3	-146.3	893.7
29	-194	896	1290	828.5	-167.2	922.8	-153.4	936.6	-156.4	933.6	-153.8	936.2
30	-160	990	1150	759.7	-174.3	975.7	-174.1	975.6	-162.2	987.8	-161.2	988.8
31	-149	1061	1210	722.9	-180.4	1029.6	-190.7	1019.3	-167.0	1043.0	-168.2	1041.8
32	-196	1084	1280	693.2	-185.4	1094.6	-200.3	1079.7	-171.8	1108.2	-175.0	1105.0
33	-242	1118	1360	651.5	-189.1	1170.9	-202.3	1157.7	-177.2	1182.8	-181.6	1178.4
34	-243	1207	1450	605.0	-191.5	1258.5	-200.8	1249.2	-181.8	1268.2	-187.8	1262.2
35	-152	1398	1550	557.8	-192.3	1357.7	-198.8	1351.2	-184.7	1365.3	-193.7	1356.3
36	-140	1530	1670	508.2	-191.4	1478.6	-198.1	1471.9	-185.0	1485.0	-199.5	1470.5
37	-218	1592	1810	464.5	-188.6	1621.4	-198.7	1611.3	-182.1	1627.9	-205.1	1604.9
38	-255	1705	1960	430.9	-183.9	1774.1	-193.9	1766.1	-176.7	1783.3	-210.6	1749.4
39	-173	1907	2140	403.0	-177.0	1963.0	-173.1	1966.9	-169.6	1970.4	-216.0	1924.0
40	-104	2246	2350	372.4	-167.9	2182.1	-131.3	2218.7	-162.5	2187.5	-221.4	2128.6

B. *Comparisons of the graduation results*

For easy comparisons, we have plotted the graduated values against ungraduated values in Figure 1.

We can note the following comparisons:

a. smoothness

We note that the cross-validation regression polynomial and basis spline methods show the underlying cubics, while the moving weighted average minimum R_z^2 and supersmoother methods reflect more the local variations. Of the latter two methods, the minimum R_z^2 fit the end values, $i = 2, 40$, rather too closely, and the supersmoother has a global shape that resembles the first two methods.

b. weighted residues

The weighted residuals $w_i(y_i - \widehat{y}_i)^2$ for them are plotted in Figure 2, where we provide a fitted curve for easy comparison. The residuals do not show any unusual patterns. The curves seem to be smooth and centre around 0.

It may be of interest to note that the four methods use different minimizing criteria. There is no priori knowledge which method will give the minimum mean weighted sum of squares of residuals or the minimum cross-validation score. It is easy to compute the mean weighted sum of squares of residuals,

$$\text{MWSSR} = \frac{1}{39} \sum_{i=2}^{40} w_i(y_i - \widehat{y}_i)^2 .$$

To compute the cross-validation score, which is

$$\text{CVS} = \frac{1}{39} \sum_{i=2}^{40} w_i(y_i - \widehat{y}_i^{(i)})^2 ,$$

with $\widehat{y}_i^{(i)}$ as the least squares predictor of y_i using only 38 data points while excluding (x_i, y_i), is not as easy. This can be accomplished, however, by its other form, see *Craven / Wahba* (1979),

$$\text{CVS} = \frac{1}{39} \sum_{i=2}^{40} \frac{w_i(y_i - \widehat{y}_i)^2}{(1 - a_{ii})^2} ,$$

Figure 1

Smoothed curves fitted to $y_i = u_i - b_i$ at ages $x = 2$ to 40

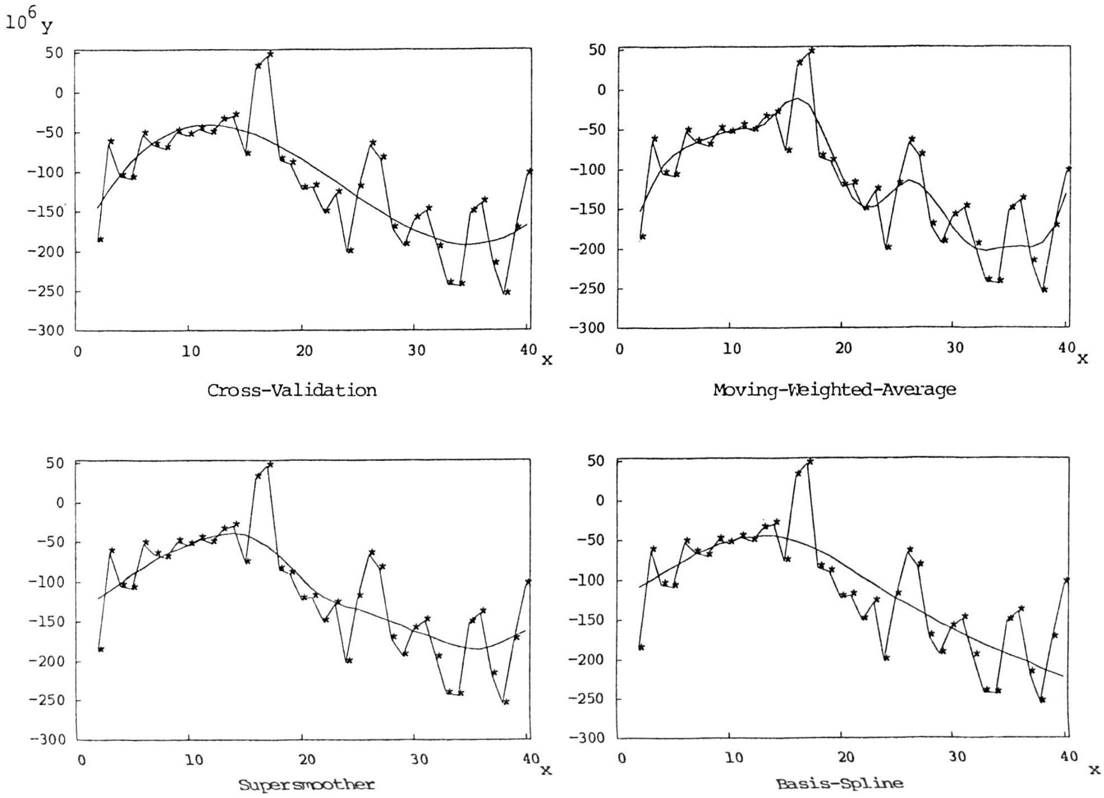
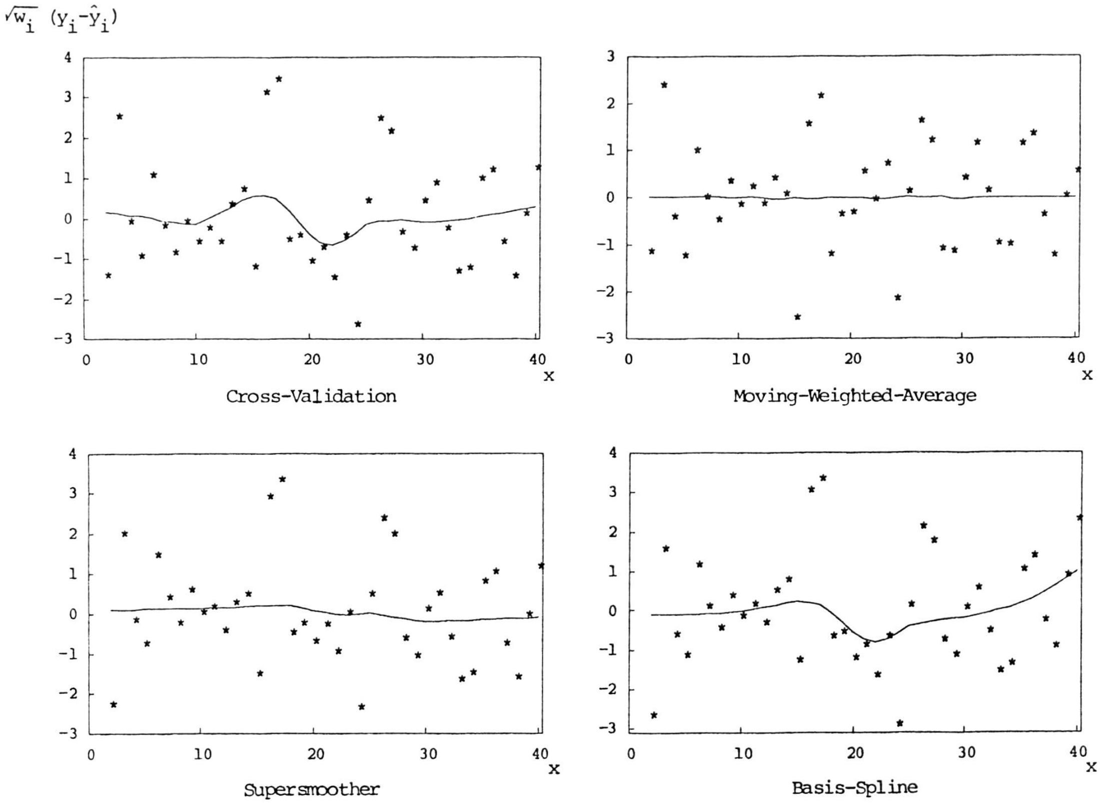


Figure 2

Supersmoother fits to the weighted residuals $\sqrt{w_i}(y_i - \hat{y}_i)$ obtained by the four methods



where a_{ii} are the diagonal elements of A, a 39 by 39 matrix which transforms $y_2, \ldots, y_{40}$ to $\hat{y}_2, \ldots, \hat{y}_{40}$.
For the weighted moving average minimum R_z^2 method, we use a_0 for a_{ii}. For the supersmoother method, we approximate a_{ii} by $1/J$, where J is 7 which is the average variable span. For the basis-spline method, a_{ii} is automatically calculated by the package. For the cross validation regression method, we make use a matrix X which is a 39 by 4 matrix, with rows $[1 \ x_i \ x_i^2 \ x_i^3]$ for different i. Then a_{ii} are the diagonal elements of $X(XX^t)^{-1}X^t$, where t means transpose. We give these calculations in Table 2.

Table 2

The mean weighted sum of squares of residuals MWSSR and cross validation score CVS

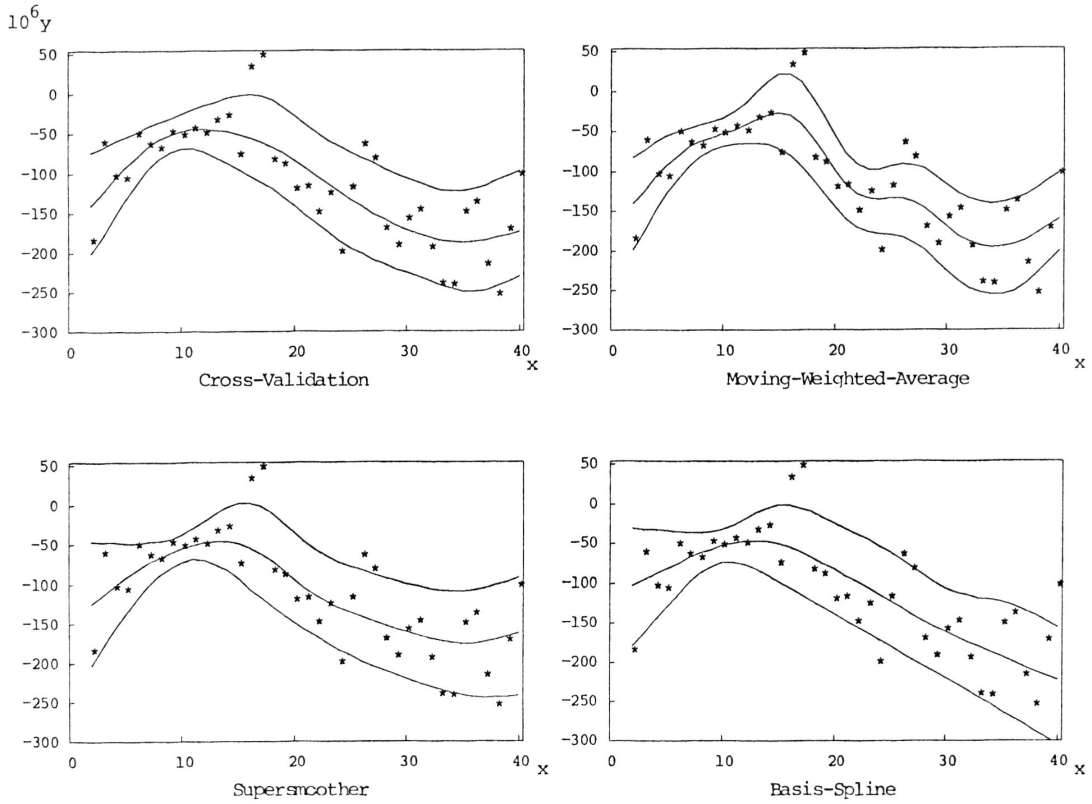
	Graduation Methods			
	Cross-Validation	Moving-Weighted-Average	Super-smoother	Basis-Spline
MWSSR	1.76817	1.18881	1.65955	1.77805
CVS	2.25892	2.05116	2.25883	2.33892

C. *Observation on the confidence intervals*

We use the bootstrap percentile method to produce the confidence intervals for the point estimates obtained from the four graduation methods. They are given in Figure 3.
It is of interest to note that the interval bands follow the curves of the point estimates; this feature is especially prominent for the moving weighted average minimum R_z^2 method.
There does not seem to be much difference in the interval lengths among the different methods. The interval lengths are shortest in the age range 8 – 12, where the original data points are clustered together. It can also be noted that most methods produce wide confidence intervals towards the end of the age range, except for the minimum R_z^2 method which has fabricated extended values for graduation.
In terms of coverage accuracy, there does not seem to be much difference either: There are four original data points that none of the confidence intervals

Figure 3

Bootstrap confidence intervals for true rates (* denotes an ungraduate value)



covers; each method has also two or three near misses. There is no clear winner or loser. On the whole, the coverage is between 32 / 39 and 35 / 39, i.e. about 82 % and 90 %.

4 Concluding remarks

We have compared four graduation methods on a common data set. Besides small differences, there does not seem to be vast differences. The conformity seems to extend to their confidence intervals. One conclusion of this paper may be that it does not matter much which graduation method one would employ, provided the graduation method's characteristics are known and the method is ready to use. For instance, one may argue that although the minimum R_z^2 method tends to reflect more local variations than other methods the method is very easy to apply.

In a sequel paper we will use the other bootstrap methods to produce confidence intervals. Recent bootstrap papers have succeeded to compare the various bootstrap methods in a uniform theoretical framework, discussing their critical points, confidence interval lengths, coverage accuracy and dependency on sample sizes. For instance, it has been shown that the bias corrected accelerated method and bootstrap method have some nice theoretical properties.

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Summary

Bootstrap method is applied to produce confidence intervals for the four following graduation methods: moving-weighted-average minimum-R_z^2, cross validation polynomial regression, supersmoother and basis-spline. The graduation methods are compared and we observe how the underlying graduation methods affect the confidence intervals in terms of coverage accuracy and interval length.

Zusammenfassung

Anhand der Bootstrap-Methode werden Konfidenzintervalle für die folgenden Methoden der Ausgleichsrechnung konstruiert: Gleitendes gewichtetes Mittel (Minimum R_z^2), polynomiale Regression mit Cross-Validierung, «Supersmoother» und «Basis-Spline». Die Ausgleichsmethoden und die entsprechenden Konfidenzintervalle werden anhand eines praktischen Beispiels verglichen.

Résumé

La méthode «bootstrap» est utilisée pour produire des intervalles de confiance dans le cas de quatre méthodes de lissage: moyenne mobile (minimum R_z^2), régression polynômiale avec validation en croix, «supersmoother» et «basis-spline». Ces méthodes de lissage et les intervalles de confiance correspondants sont comparés à l'aide d'un exemple pratique.