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Hierarchical Models in Insurance

1 Introduction

In a "chain of reinsurance", every company C_i buys reinsurance from company C_{i+1} and sells to company C_{i-1}. C_0 is only a buyer and C_n, at the end of the chain, is only a seller. Some optimal risk exchanges between the companies of the chain were calculated by *Gerber* (1984). *Lemaire/Quairière* (1986) used Pareto-optimality to find the best transactions in the chain.
In reality, one company of course has more than one client. Therefore, *Heijnen* (1989) introduced a tree-hierarchical structure, in which one company can have many clients. To get explicit results, Heijnen didn't make restrictions on the utility functions (as usual), but on the "nature" of the risk; every risk X had to be "small", i.e. rather stable around its mean $E(X)$. In practice, one could use the results e.g. for life insurance portfolios, which have rather stable results every year.
For other types of risks, one has to choose explicitly a utility function, to find an analytic solution. The most usual choice is then to take exponential utilities:

$$ {}_ju_i(x) = \frac{1}{{}_ja_i}[1 - e^{-{}_ja_i x}], \qquad -\infty < x < \infty \tag{1.1}$$

for company i on tree-level j (see section 2 for more details on the indices). The coefficient ${}_ja_i$ can be interpreted as the "risk aversion" of the company and is equal to

$$ -\frac{{}_ju_i''(x)}{{}_ju_i'(x)}, \qquad -\infty < x < \infty. $$

Another disadvantage of the approximations used by Heijnen (1989) is that results are only determined up to some arbitrary constant c, which is in fact the expected value of the result. This means that one first has to choose a "desired level of average yearly reimbursement", before determining the optimal risk exchange.

In this papier we start from another point of view. We consider a number of direct insurers (level 1 in the tree) with surplusses ${}_1Z_i$. Every company i has accepted to insure the risk X_i, and is looking for reinsurance covering for that risk on the second level of the tree. Its modified surplus after reinsurance is ${}_1Y_i$. The reinsurance companies in the tree (on levels $j = 2$ until n) have surplusses ${}_jZ_i$, before reinsuring their parts of the original risks X_k. After accepting the reinsurance, their surplusses will have changed into ${}_jY_i$. We will calculate explicit expressions for those modified surplusses, using the concept of "Pareto-optimal risk exchanges", introduced by *Borch* (1962)., An exchange is said to be Pareto-optimal if it cannot be improved for all companies at the same time. In our models we need the famous "theorem of *Borch*" on Pareto-optimal risk exchanges. It says fhe following:

Borch's theorem

For n insurance companies with utility functions $u_1, \dots, u_n$ and surplusses $Z_1, \dots, Z_n$ respectively, the following statements are equivalent:

a) A risk exchange $(Y_1, \dots, Y_n)$ is Pareto-optimal
b) Some positive constants $c_2, \dots, c_n$ exist, such that

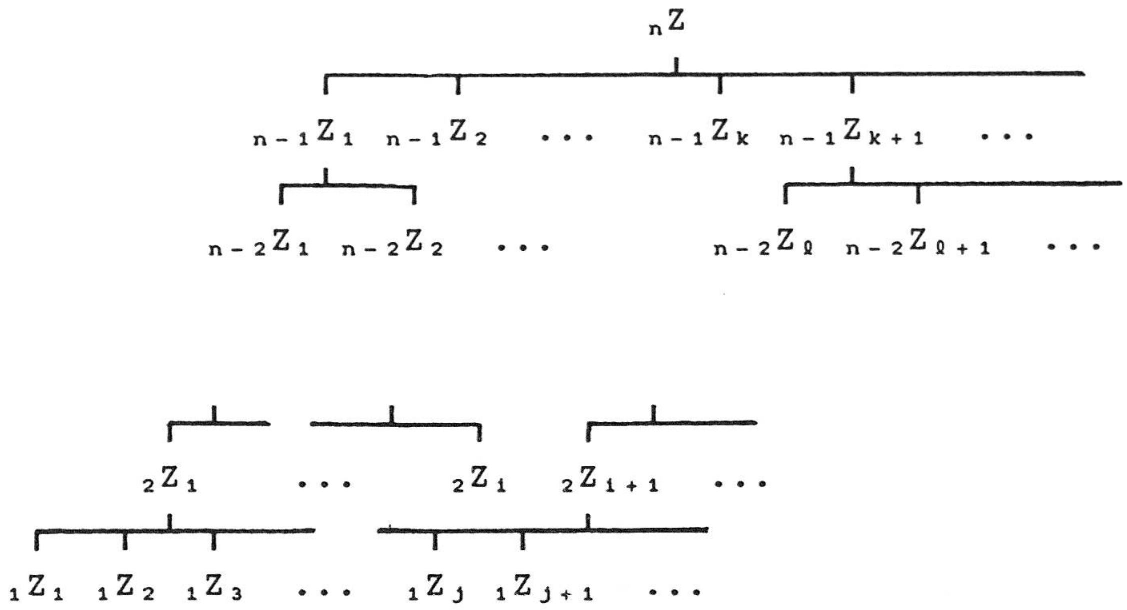
$$u_j'(Y_j) = c_j u_1'(Y_1), \qquad \text{for} \quad j = 2, \dots, n. \tag{1.2}$$

This theorem will be used between successive levels in the tree. A proof can be found, e.g. in *Bühlmann* (1970).

2 Construction of the tree

We will use the same kind of notations as *Heijnen* (1989) introduced in his paper. A company on level j sells a reimbursement to several companies on level $j-1$. Each of those companies is in touch with several companies on level $j-2$ and so on. On each level we number the companies (and their stochastic variables) with a right-under-index, while a left- under-index denotes the level itself. The tree which we consider, consists of n levels. At the top, we only have one company, with surplus ${}_nZ$. Every other level has many companies, and we will use ${}_jZ_i$ for the initial surplus of the i-th company on level j. Finally we also need a notation for the sum of the surplusses of the companies on the k-th level connected with company i on level j (with of course $k < j$).

Therefore we will use the notation ${}^{j}_{k}Z^{i}_{\cdot}$. As an illustration, we present a picture of such a tree in the following figure.



In the following sections, we will always use ${}_jZ_i$ as notation for the initial surplus of company i on level j, while ${}_jY_i$ denotes its surplus after the risk transaction. Furthermore, we assume that this company evaluates its surplus with an exponential utility function ${}_ju_i(x)$, described in (1.1), with risk aversion ${}_ja_i$.

3 Forward Pareto-optimal model

In this section and in section 4, we suppose that company i on level j buys a reimbursement ${}_jR_i$ for a premium ${}_j\pi_i$. It will not be necessary to know a priori how to calculate the premium; assuming the utility functions to be exponential will be enough to determine the optimal ${}_jR_i$. Afterwards, one can apply any premium calculation principle on this optimal ${}_jR_i$.
In this forward model, we only have local Pareto-optimality. Indeed, when working out the transaction between company j on level ℓ and company i on level $\ell+1$, we don't know whether or not the second one looks for a reinsurer on level $\ell+2$ to improve its situation.

Suppose company j on level 1 is in contact with company i on level 2. We can describe the transaction as

$$\begin{aligned} {}_1Y_j &= {}_1Z_j - {}_1\pi_j + {}_1R_j \\ {}_2Y_1' &= {}_2Z_i + {}_1^2\pi^i_{.} - {}_1^2R^i_{.} \end{aligned} \tag{3.1}$$

and this will be the same for every transaction between the levels 1 and 2. Every reimbursement ${}_1R_j$ will be determined as a Pareto-optimal solution by using *Borch's* theorem. The accent $({}_2Y_1')$ denotes the fact that the situation of this company is not yet final, because, after selling reinsurance to a company of the first level, it will look for protection at some company on the third level. This will change its surplus form ${}_2Y_i'$ into ${}_2Y_i$.
In general, the risk exchange between company j on level ℓ and company i on level $\ell+1$ can be described as follows:

$$\begin{aligned} {}_\ell Y_j &= {}_\ell Z_j + {}_{\ell-1}^{\ell}\pi^j_{.} - {}_{\ell-1}^{\ell}R^j_{.} - {}_\ell\pi_j + {}_\ell R_j \\ {}_{\ell+1}Y_i' &= {}_{\ell+1}Z_i + {}_\ell^{\ell+1}\pi^i_{.} - {}_\ell^{\ell+1}R^i_{.}. \end{aligned} \tag{3.2}$$

This holds for each company on level ℓ connected with a company on level $\ell+1$. Again, every ${}_\ell R_j$ is determined as a Pareto-optimal solution using *Borch's* theorem.
According to this theorem, the transaction leads to one equation (we have a risk exchange between two companies)

$$\begin{aligned} &{}_{\ell+1}u_i'\,({}_{\ell+1}Z_i + {}_\ell^{\ell+1}\pi^i_{.} - {}_\ell^{\ell+1}R^i_{.}) \\ &= {}_\ell^{\ell+1}k_j^i\,{}_\ell u_j'\,({}_\ell Z_j + {}_{\ell-1}^{\ell}\pi^j_{.} - {}_{\ell-1}^{\ell}R^j_{.} - {}_\ell\pi_j + {}_\ell R_j) \end{aligned} \tag{3.3}$$

with ${}_\ell^{\ell+1}k_j^i$ a positive constant.
Since a claim equal to zero implies a reimbursement equal to zero, also the equation

$${}_{\ell+1}u_i'\,({}_{\ell+1}Z_i + {}_\ell^{\ell+1}\pi^i_{.}) = {}_\ell^{\ell+1}k_j^i\,{}_\ell u_j'\,({}_\ell Z_j + {}_{\ell-1}^{\ell}\pi^j_{.} - {}_\ell\pi_j) \tag{3.4}$$

must hold. This will eliminate ${}_\ell^{\ell+1}k_j^i$ in (3.3), and we get

$$\begin{aligned} &\frac{{}_{\ell+1}u_i'\,({}_{\ell+1}Z_i + {}_\ell^{\ell+1}\pi^i_{.} - {}_\ell^{\ell+1}R^i_{.})}{{}_{\ell+1}u_i'\,({}_{\ell+1}Z_i + {}_\ell^{\ell+1}\pi^i_{.})} \\ &= \frac{{}_\ell u_j'\,({}_\ell Z_j + {}_{\ell-1}^{\ell}\pi^j_{.} - {}_{\ell-1}^{\ell}R^j_{.} - {}_\ell\pi_j + {}_\ell R_j)}{{}_\ell u_j'\,({}_\ell Z_j + {}_{\ell-1}^{\ell}\pi^j_{.} - {}_\ell\pi_j)}. \end{aligned} \tag{3.5}$$

Using exponential utility functions, as described in formula (1.1), this equation can be written as

$$\frac{\exp[-_{\ell+1}a_i\,({}_{\ell+1}Z_i + {}^{\ell+1}_{\ell}\pi^i_{.} - {}^{\ell+1}_{\ell}R^i_{.})]}{\exp[-_{\ell+1}a_i\,({}_{\ell+1}Z_i + {}^{\ell+1}_{\ell}\pi^i_{.})]} = \frac{\exp[-_{\ell}a_j\,({}_{\ell}Z_j + {}^{\ell}_{\ell-1}\pi^j_{.} - {}^{\ell}_{\ell-1}R^j_{.} - {}_{\ell}\pi_j + {}_{\ell}R_j)]}{\exp[-_{\ell}a_j\,({}_{\ell}Z_j + {}^{\ell}_{\ell-1}\pi^j_{.} - {}_{\ell}\pi_j)]} \tag{3.6}$$

which simplifies to

$${}_{\ell+1}a_i\,{}^{\ell+1}_{\ell}R^i_{.} = {}_{\ell}a_j\,({}^{\ell}_{\ell-1}R^j_{.} - {}_{\ell}R_j). \tag{3.7}$$

We introduce the notation

$${}^{m}_{\ell}p^i_j = \frac{{}_{m}a_i}{{}_{\ell}a_j}. \tag{3.8}$$

Then we can write (3.7) as

$${}_{\ell}R_j = {}^{\ell}_{\ell-1}R^j_{.} - {}^{\ell+1}_{\ell}p^i_j\,{}^{\ell+1}_{\ell}R^i_{.}. \tag{3.9}$$

After summation over all j (i.e. over all companies on level ℓ connected with company i on level $\ell+1$) we get

$${}^{\ell+1}_{\ell}R^i_{.} = {}^{\ell+1}_{\ell-1}R^i_{.} - {}^{\ell+1}_{\ell}p^i_{.}\,{}^{\ell+1}_{\ell}R^i_{.}$$

or

$${}^{\ell+1}_{\ell}R^i_{.} = \frac{1}{1 + {}^{\ell+1}_{\ell}p^i_{.}}\,{}^{\ell+1}_{\ell-1}R^i_{.}. \tag{3.10}$$

Substituting (3.10) in (3.9), we find

$${}_{\ell}R_j = {}^{\ell}_{\ell-1}R^j_{.} - \frac{{}^{\ell+1}_{\ell}p^i_j}{1 + {}^{\ell+1}_{\ell}p^i_{.}}\,{}^{\ell+1}_{\ell-1}R^i_{.}. \tag{3.11}$$

In particular, we have for the optimal reimbursements of the first level

$${}_{1}R_j = {}^{1}_{0}R^j_{.} - \frac{{}^{2}_{1}p^i_j}{1 + {}^{2}_{1}p^i_{.}}\,{}^{2}_{0}R^i_{.}$$

or

$$
{}_1R_j = X_j - \frac{{}^2_1p^i_j}{1 + {}^2_1\,p^i_.}\,{}^2X^i_. \tag{3.12}
$$

with ${}^2X^i_.$ the sum of the original risks X_k of companies on level 1 connected with company i on level 2.
In formula (3.11) we see that the reimbursements of the companies on level ℓ depend on those of the companies on level $\ell - 1$. This fact, used in an inductive way, and combined with (3.12), shows us that every reimbursement ${}_\ell R_j$ depends on the original risks X_k.
We are also able to find a result for the "net" payment (i.e. reimbursements sold minus reimbursement bought)

$$
{}_\ell D_j = {}^{\ell}_{\ell-1}R^j_. - {}_\ell R_j.
$$

This payment is equal to

$$
{}_\ell D_j = \frac{{}^{\ell+1}_{\ell}p^i_j}{1 + {}^{\ell+1}_{\ell}p^i_.}\,{}^{\ell+1}_{\ell-1}R^i_. \tag{3.13}
$$

These two results, (3.11) and (3.13), are very similar to those found by *Heijnen* (1989), who used general utility functions but assumed the risk to be stable around its mean.

4 Backward Pareto-optimal model

In stead of local Pareto-optimality, used in our first model, we now assume the tree to be entirely formed. The different transactions will use global Pareto-optimality. Indeed, in this model a company on level ℓ will make the amount of reimbursements it sells to companies on level $\ell - 1$, depending on the reinsurance it can buy form a company on level $\ell + 1$.
Suppose company j on level 1 looks for protection with company i on level 2. Their transaction can be described as follows:

$$
\begin{aligned}
{}_1Y_j &= {}_1Z_j - {}_1\pi_j + {}_1R_j \\
{}_2Y_i &= {}_2Z_i + {}^2_1\pi^i_. - {}^2_1R^i_. - {}_2\pi_i + {}_2R_i
\end{aligned} \tag{4.1}
$$

and this is true for all transactions between the levels 1 and 2. The reimbursements ${}_1R_j$ will be determined as Pareto-optimal solutions by *Borch's* theorem, and will depend on ${}_2R_i$ which is the reimbursement company i itself can buy from a higher level.
More general, the risk exchange between company j on level ℓ and company i on level $\ell + 1$ leads to the situation

$$\begin{aligned} {}_{\ell}Y_j &= {}_{\ell}Z_j + {}^{\ell}_{\ell-1}\pi^j_{\cdot} - {}^{\ell}_{\ell-1}R^j_{\cdot} - {}_{\ell}\pi_j + {}_{\ell}R_j \\ {}_{\ell+1}Y_i &= {}_{\ell+1}Z_i + {}^{\ell+1}_{\ell}\pi^i_{\cdot} - {}^{\ell+1}_{\ell}R^i_{\cdot} - {}_{\ell+1}\pi_i + {}_{\ell+1}R_i \end{aligned} \tag{4.2}$$

with ${}_{\ell}R_j$ the Pareto-optimal solution, found by *Borch's* theorem.
Using this theorem on the last situation, we find the equation

$$\begin{aligned} &{}_{\ell+1}u_i'\left({}_{\ell+1}Z_i + {}^{\ell+1}_{\ell}\pi^i_{\cdot} - {}^{\ell+1}_{\ell}R^i_{\cdot} - {}_{\ell+1}\pi_i + {}_{\ell+1}R_i\right) \\ &\quad = {}^{\ell+1}_{\ell}k^i_j \, {}_{\ell}u_j'\left({}_{\ell}Z_j + {}^{\ell}_{\ell-1}\pi^j_{\cdot} - {}^{\ell}_{\ell-1}R^j_{\cdot} - {}_{\ell}\pi_j + {}_{\ell}R_j\right) \end{aligned} \tag{4.3}$$

and also

$$\begin{aligned} &{}_{\ell+1}u_i'\left({}_{\ell+1}Z_i + {}^{\ell+1}_{\ell}\pi^i_{\cdot} - {}_{\ell+1}\pi_i\right) \\ &\quad = {}^{\ell+1}_{\ell}k^i_j \, {}_{\ell}u_j'\left({}_{\ell}Z_j + {}^{\ell}_{\ell-1}\pi^j_{\cdot} - {}_{\ell}\pi_j\right) \end{aligned} \tag{4.4}$$

since the reimbursement will be equal to zero when the claim equals zero.
Thus

$$\begin{aligned} &\frac{{}_{\ell+1}u_i'\left({}_{\ell+1}Z_i + {}^{\ell+1}_{\ell}\pi^i_{\cdot} - {}^{\ell+1}_{\ell}R^i_{\cdot} - {}_{\ell+1}\pi_i + {}_{\ell+1}R_i\right)}{{}_{\ell+1}u_i'\left({}_{\ell+1}Z_i + {}^{\ell+1}_{\ell}\pi^i_{\cdot} - {}_{\ell+1}\pi_i\right)} \\ &\quad = \frac{{}_{\ell}u_j'\left({}_{\ell}Z_j + {}^{\ell}_{\ell-1}\pi^j_{\cdot} - {}^{\ell}_{\ell-1}R^j_{\cdot} - {}_{\ell}\pi_j + {}_{\ell}R_j\right)}{{}_{\ell}u_j'\left({}_{\ell}Z_j + {}^{\ell}_{\ell-1}\pi^j_{\cdot} - {}_{\ell}\pi_j\right)} \end{aligned} \tag{4.5}$$

Assuming exponential utility functions, we obtain

$$\begin{aligned} &\frac{\exp\left[-{}_{\ell+1}a_i\left({}_{\ell+1}Z_i + {}^{\ell+1}_{\ell}\pi^i_{\cdot} - {}^{\ell+1}_{\ell}R^i_{\cdot} - {}_{\ell+1}\pi_i + {}_{\ell+1}R_i\right)\right]}{\exp\left[-{}_{\ell+1}a_i\left({}_{\ell+1}Z_i + {}^{\ell+1}_{\ell}\pi^i_{\cdot} - {}_{\ell+1}\pi_i\right)\right]} \\ &\quad = \frac{\exp\left[-{}_{\ell}a_j\left({}_{\ell}Z_j + {}^{\ell}_{\ell-1}\pi^j_{\cdot} - {}^{\ell}_{\ell-1}R^j_{\cdot} - {}_{\ell}\pi_j + {}_{\ell}R_j\right)\right]}{\exp\left[-{}_{\ell}a_j\left({}_{\ell}Z_j + {}^{\ell}_{\ell-1}\pi^j_{\cdot} - {}_{\ell}\pi_j\right)\right]} \end{aligned} \tag{4.6}$$

which simplifies to

$${}_{\ell+1}a_i\left({}^{\ell+1}_{\ell}R^i_{\cdot} - {}_{\ell+1}R_i\right) = {}_{\ell}a_j\left({}^{\ell}_{\ell-1}R^j_{\cdot} - {}_{\ell}R_j\right), \tag{4.7}$$

which means that ${}_{\ell}a_j\,({}_{\ell-1}^{\ \ell}R^j_{\cdot} - {}_{\ell}R_j)$ is independent of ℓ and j! We will denote this common quantity by H. So we get

$$\sum_{\ell=1}^{n}\sum_{j}\left(\frac{H}{{}_{\ell}a_j}\right) = \sum_{\ell=1}^{n}\sum_{j}({}_{\ell-1}^{\ \ell}R^j_{\cdot} - {}_{\ell}R_j) = X \tag{4.8}$$

with X the sum of the original risks X_k, and with the summation taken over all companies of the tree.
We introduce the notation

$$\frac{1}{:a.} = \sum_{\ell=1}^{n}\sum_{j}\frac{1}{{}_{\ell}a_j} \tag{4.9}$$

which gives us

$$H =: a.X \tag{4.10}$$

The "net" payment

$${}_{\ell}D_j = {}_{\ell-1}^{\ \ell}R^j_{\cdot} - {}_{\ell}R_j \tag{4.11}$$

then becomes

$${}_{\ell}D_j = \frac{:a.}{{}_{\ell}a_j}X \tag{4.12}$$

This result will enable us to get explicit results for the reimbursements ${}_{\ell}R_j$.
We have

$${}_{\ell-1}D_k = {}_{\ell-2}^{\ell-1}R^k_{\cdot} - {}_{\ell-1}R_k.$$

Summing over all k, connected with company j on level ℓ, one finds

$${}_{\ell-1}^{\ \ell}D^j_{\cdot} = {}_{\ell-2}^{\ \ell}R^j_{\cdot} - {}_{\ell-1}^{\ \ell}R^j_{\cdot}$$

and substituting this in (4.11), we get

$${}_{\ell}D_j + {}_{\ell-1}^{\ \ell}D^j_{\cdot} = {}_{\ell-2}^{\ \ell}R^j_{\cdot} - {}_{\ell}R_j. \tag{4.13}$$

Next, we have

$$_{\ell-2}D_k = {}^{\ell-2}_{\ell-3}R^k_{\cdot} - {}_{\ell-2}R_k\,.$$

Summing again over all k, connected with company j on level ℓ, one finds

$${}_{\ell-2}^{\ell}D^j_{\cdot} = {}_{\ell-3}^{\ell}R^j_{\cdot} - {}_{\ell-2}^{\ell}R^j_{\cdot}$$

and substituting this in (4.13), we get

$${}_{\ell}D_j + {}_{\ell-1}^{\ell}D^j_{\cdot} + {}_{\ell-2}^{\ell}D^j_{\cdot} = {}_{\ell-3}^{\ell}R^j_{\cdot} - {}_{\ell}R_j\,. \tag{4.14}$$

Generalization of this result gives us

$${}_{\ell}R_j = {}^{\ell}X^j_{\cdot} - ({}_{\ell}D_j + {}_{\ell-1}^{\ell}D^j_{\cdot} + \cdots + {}_{1}^{\ell}D^j_{\cdot})$$

with ${}^{\ell}X^j_{\cdot}$ the sum of the original risks X_k of companies on level 1, connected (undirectly) with company j on level ℓ. Using (4.12), we can write this as

$${}_{\ell}R_j = {}^{\ell}X^j_{\cdot} - \,:a.\left(\frac{1}{{}_{\ell}a_j} + \sum_i \frac{1}{{}_{\ell-1}a_i} + \cdots + \sum_i \frac{1}{{}_{1}a_i}\right)X.$$

We introduce the notation

$$\frac{1}{{}_{\ell}a_j^+} = \sum_{k=1}^{\ell-1}\sum_i \frac{1}{{}_{k}a_i} \tag{4.15}$$

with the summation over i taken over those companies on level k which are in touch (undirectly) with company j on level ℓ. So we get

$${}_{\ell}R_j = {}^{\ell}X^j_{\cdot} - \,:a.\left(\frac{1}{{}_{\ell}a_j} + \frac{1}{{}_{\ell}a_j^+}\right)X. \tag{4.16}$$

This result can be transformed into an expression which is more similar to the solutions obtained by *Heijnen* (1989) using the following notation

$${}_{\ell}p_j^+ = \sum_{k=1}^{\ell-1}\sum_i {}_{k}^{n}p_i \tag{4.17}$$

where the summations are the same as in (4.15), and where ${}^{n}_{k}p_i = {}_{n}a/{}_{k}a_i$ as described in (3.8).
In this case we have

$$ {}_{n}a\left(\frac{1}{{}_{\ell}a_j} + \frac{1}{{}_{\ell}a_j^+}\right) = {}^{n}_{\ell}p_j + {}_{\ell}p_j^+ \tag{4.18} $$

and also

$$ {}_{n}a\frac{1}{:a.} = 1 + {}_{n}p^+. \tag{4.19} $$

The result of (4.16) can now be rewritten as

$$ {}_{\ell}R_j = {}^{\ell}X^j_{.} - \frac{{}^{n}_{\ell}p_j + {}_{\ell}p_j^+}{1 + {}_{n}p^+}X \tag{4.20} $$

Which is the final expression for the reimbursement bought by company j on level ℓ from a company one level higher in the tree.

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Summary

In this paper we present a generalization of the "chain of reinsurance" – model, first introduced by *Gerber*, and developed further by a.o. *Lemaire/Quairière*. In a chain, every company is linked to only one other company, as if it would have just one client. To describe real situations however, one needs a kind of "tree" structure, where one company is in contact with several clients. Using exponential utilities, we calculate the optimal risk exchange for several optimality criteria.

Zusammenfassung

In der vorliegenden Arbeit wird eine Verallgemeinerung des zuerst von *Gerber* eingeführten und von *Lemaire/Quairière* weiterentwickelten Modells von "Rückversicherungsketten" vorgestellt. In einer solchen Kette ist jede Gesellschaft mit einer einzigen anderen Gesellschaft verbunden, als ob sie nur gerade einen Kunden hätte. Die Beschreibung der realen Situationen erfordert jedoch eine Baumstruktur, in der die Gesellschaften mit mehreren Klienten in Kontakt sind. Unter Zugrundelegung von exponentiellen Utilitäten wird für verschiedene Optimalitätskriterien der optimale Risikoaustausch ermittelt.

Résumé

Les auteurs présentent une généralisation de la notion de "chaîne de réassurance" introduite par *Gerber*, puis développée entre autres par *Lemaire/Quairière*. Dans une chaîne chaque compagnie est liée à une seule autre compagnie comme si elle n'avait qu'un seul client. Les situations réelles nécessitent de disposer d'une structure arborescente dans laquelle les compagnies sont en contact avec plusieurs clients. Sur la base d'utilités exponentielles, les auteurs déterminent les échanges de risque optimaux correspondant à divers critères d'optimalité.