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On excess of loss reinsurance with reinstatements

1 Introduction

The topic of the present paper is calculation of premiums for excess of loss (xl) reinsurance with reinstatements. In practical applications of xl reinsurance one often applies reinstatements. However, in the actuarial literature they are very rare; an exception is *Simon* (1972).

In Sections 2 we define some concepts from practical xl reinsurance. Section 3 is devoted to calculation of pure premiums. This theory is generalized to loaded premiums according to the standard deviation principle in Section 4. In Sections 3 and 4 we consider the distribution of the aggregate claims payments of the reinsurer as given. In Section 5 we discuss how to evaluate this distribution when the aggregate claim is generated by a compound distribution. In Section 6 we look at a special case of the model of Section 5, assuming that the number of claims is Poisson distributed and the severities are Pareto distributed. A numerical example is given.

An earlier version of the present paper was presented at the XXIInd ASTIN Colloquium in Montreux in September 1990. At the same colloquium *Bernegger* (1990) presented similar ideas independent of the present paper. However, for loaded premiums *Bernegger* applies the variance principle instead of the standard deviation principle.

2 Some concepts from practical excess of loss reinsurance

When the present author in 1989 started working with xl reinsurance in practice, he immediately discovered that the area was much more complex than the impression he had got from the actuarial literature. He was not familiar with concepts like reinstatement, aggregate deductible, and aggregate limit. As some of the readers might be in the same situation, we shall try to explain some of these concepts.

We consider an insurance portfolio during one year. Let N denote the number of claims occurred in the portfolio during the year and Y_i the size of the ith of these claims ($i = 1, \ldots, N$).

Xl reinsurance is a non-proportional reinsurance form that covers the part of each claim that exceeds a *deductible* l, that is, the reinsurer covers

$$Z_i = \max(0, Y_i - l) \tag{2.1}$$

of claim i. In practice there is also often a limit m on the payment of each claim, that is, the reinsurer covers

$$Z_i = \min(\max(0, Y_i - l), m). \tag{2.2}$$

We call this an *xl* reinsurance for the *layer m in excess of l* ($m\,xs\,l$). If $m = \infty$, we have an *unlimited* layer with deductible l.
Before continuing the presentation of reinsurance concepts, we make a notational remark. When comparing formulae (2.1) and (2.2), we see that in (2.1) we have used the symbol Z_i for a claim to the unlimited layer with deductible l whereas in (2.2) we have used the same symbol for a claim to the layer $m\,xs\,l$. To avoid confusion, we could have denoted the two quantities by e.g. respectively ${}_lZ_i$ and ${}^m_lZ_i$. However, to not overload our notation with bells and whistles, we try to keep the explicit use of indices to a minimum, hoping that this will not confuse our readers.
We assume an *xl* reinsurance for the layer $m\,xs\,l$. Let X denote the aggregate claim to the layer, that is,

$$X = \sum_{i=1}^{N} Z_i$$

with Z_i given by (2.2); we make the convention that $\sum_{i=1}^{0} = 0$. For simplicity we assume that X has finite variance. However, most of the results of the paper hold under more relaxed assumptions.
In practice there is often an *aggregate deductible* L. This means that the reinsurer covers only the part of the aggregate claim that exceeds L, that is,

$$X' = \max(0, X - L).$$

This is called an *xl* reinsurance for the layer *$m\,xs\,l\,xs\,L$ in the aggregate*. Often there is also an *aggregate limit* M, that is, there is a limit M on the aggregate claim; the reinsurance covers

$$X' = \min(\max(0, X - L), M).$$

We say that the *aggregate layer* is $M \operatorname{xs} L$. The aggregate deductible and the aggregate limit are usually given as whole multiples of the limit m. If $M = (K+1)m$, we have an *xl* reinsurance for the layer $m \operatorname{xs} L$ in the aggregate with K *reinstatements*. The idea is that the reinsurance is thought of as being for an aggregate layer of the same size as the layer for the individual claims, and thus the reinsurance has to be reinstated if the aggregate payment exceeds a whole multiple of the limit. If $M = m$, we have an *xl* reinsurance with no reinstatements.

There are several ways of paying premiums for an *xl* reinsurance with reinstatements. We shall distinguish between *free* and *paid* reinstatements. The simplest case is when all the reinstatements are free. For an *xl* reinsurance for the layer $m \operatorname{xs} l \operatorname{xs} L$ in the aggregate with K free reinstatements, we simply consider finding a fixed premium for an *xl* reinsurance for the layer $m \operatorname{xs} l$ with aggregate layer $(K+1)m \operatorname{xs} L$. With paid reinstatements the idea for the term reinstatement becomes more clear; to reinstate the layer, the ceding company has to pay a *reinstatement premium*, which in practice is given as a percentage of the premium initially paid for the layer. The premium for the next reinstatement is paid pro rata of the claims to the layer.

To clarify the ideas, let P be the initial premium and $c_k P$ the premium for the kth reinstatement; $c_k = 0$ if the kth reinstatement is free. We say that the kth reinstatement is at $100c_k$ %. This reinstatement covers

$$r_{Lk} = \min(\max(0, X - km - L), m);$$

for simplicity we call the cover of the original layer the 0th reinstatement. Then the premium to be paid for the kth reinstatement, is $c_k P r_{L,k-1}/m$. The aggregate claim payments on this reinsurance are

$$R_{LK} = \sum_{k=0}^{K} r_{Lk} = \min(\max(0, X - L), (K+1)m),$$

and the total premium income is

$$T = P\left(1 + \frac{1}{m}\sum_{k=1}^{K} c_k r_{L,k-1}\right). \tag{2.3}$$

As an alternative to deductible, we sometimes have a *franchise*. Then the reinsurer covers claims exceeding the franchise, usually up to a finite limit. An

xl reinsurance with franchise d and limit $m\,(\geq d)$ covers

$$Z_i = \begin{cases} 0 & (Y_i < d) \\ \min(Y_i, m). & (Y_i \geq d) \end{cases}$$

of the ith claim. An aggregate franchise is defined analogously to a franchise on the individual claim.
We mention that there exists a multiple of other variants of xl reinsurance that will not be covered in the present paper.

3 Pure premiums

We consider an xl reinsurance for the layer $m\,xs\,l$ and keep the notation from Section 2. We introduce the cumulative distribution G of X and its stop loss transform $\overline{G}$ given by

$$\overline{G}(t) = \int_t^\infty (x-t)\,dG(x) = \int_t^\infty (1-G(x))\,dx = \mathrm{E}\max(0, X-t),$$

cf. e.g. *Sundt* (1991, subsection 10.3). The pure premium for the aggregate layer $M\,xs\,L$ is $\overline{G}(L) - \overline{G}(L+M)$. In particular, the pure single premium for the kth reinstatement is

$$d_{Lk} = Er_{Lk} = \overline{G}(L+km) - \overline{G}(L+(k+1)m).$$

We introduce the cumulative quantity

$$D_{Lk} = ER_{Lk} = \sum_{i=0}^{k} d_{Li} = \overline{G}(L) - \overline{G}(L+(k+1)m),$$

which is the pure premium for an xl reinsurance for the layer $m\,xs\,l\,xs\,L$ in the aggregate with k free reinstatements. By letting k approach infinity, we obtain

$$R_{L\infty} = \sum_{i=0}^{\infty} r_{Li} = \max(0, X-L)$$

$$D_{L\infty} = ER_{L\infty} = \sum_{i=0}^{\infty} d_{Li} = \overline{G}(L);$$

In particular, $D_{0\infty} = EX$.

Our reinsurance is assumed to have K reinstatements, that is, $M = (K+1)m$. For the 0th reinstatement the premium is P. The kth reinstatement is at $100c_k$ %. The pure premiums should satisfy

$$ET = ER_{LK}$$

with T given by (2.3), that is,

$$P\left(1+\frac{1}{m}\sum_{k=1}^{K} c_k d_{L,k-1}\right) = D_{LK}. \tag{3.1}$$

We see that (3.1) gives some sort of global equilibrium; the expected premium income should be equal to the expected claim payments. Using this idea on each reinstatement, that is, the expected reinstatement premium should be equal to the expected claim payments on that reinstatement, we obtain

$$P = d_{L0} \tag{3.2}$$
$$P c_k d_{L,k-1}/m = d_{Lk}, \quad (k = 1, 2, \dots, K)$$

that is,

$$c_k = \frac{m d_{Lk}}{P d_{L,k-1}}. \quad (k = 1, 2, \dots, K) \tag{3.3}$$

In practice, the reinstatement percentages c_k are usually fixed in advance, usually equal to 100 % or 50 % (or 0 % if the reinstatement is free). If the c_k's are fixed in advance, we solve (3.1) for P to obtain

$$P = \frac{D_{LK}}{1+\frac{1}{m}\sum_{k=1}^{K} c_k d_{L,k-1}}. \tag{3.4}$$

If all reinstatements are at $100c$ %, then (3.4) reduces to

$$P = \frac{D_{LK}}{1+c\frac{D_{L,K-1}}{m}}.$$

By letting $K \to \infty$ we obtain

$$P_\infty = \lim_{K\to\infty} P = \frac{D_{L\infty}}{1+c\frac{D_{L\infty}}{m}}.$$

In particular, if $L = 0$, that is, no aggregate deductible, we have

$$P_\infty = \frac{EX}{1 + c\frac{EX}{m}}.$$

4 Loaded premiums

Let U be an arbitrary risk. By the standard deviation principle, the premium for this risk is

$$P_U = EU + \gamma\sqrt{\operatorname{Var} U}$$

for some loading γ (cf. e.g. *Sundt* (1991), Chapter 4). In Section 4 we shall apply this principle to the pro rata payment of reinstatement premiums described in Section 2. Unfortunately it is not obvious how to proceed in this case as the premium income itself is a random quantity correlated with the claim payments on the reinstatement.
We shall need the quantities

$$v_{Lij} = \operatorname{Cov}(r_{Li}, r_{Lj}) \qquad V_{Lk} = \operatorname{Var} R_{Lk}.$$
$$v_{Lk} = v_{Lkk} = V_{L+km,1} = \operatorname{Var} r_{Lk} \quad w_{Lk} = v_{L,k-1,k}.$$

We have

$$\begin{aligned} V_{Lk} &= \operatorname{Var} R_{Lk} = ER_{Lk}^2 - E^2 R_{Lk} \\ &= \int_L^{L+(k+1)m} (x-L)^2 dG(x) + (k+1)^2 m^2 (1 - G(L+(k+1)m)) - D_{Lk}^2 \\ &= \overline{\overline{G}}(L) - \overline{\overline{G}}(L+(k+1)m) - 2(L+(k+1)m)\overline{G}(L+(k+1)m) - D_{Lk}^2 \end{aligned}$$

with

$$\overline{\overline{G}}(t) = \int_t^\infty (x-t)^2\, dG(x).$$

As a special case we obtain

$$v_{Lk} = \overline{\overline{G}}(L+km) - \overline{\overline{G}}(L+(k+1)m) - 2m\overline{G}(L+(k+1)m) - d_{Lk}^2.$$

For $j > i$,

$$v_{Lij} = \operatorname{Cov}(r_{Li}, r_{Lj}) = Er_{Li}r_{Lj} - Er_{Li}Er_{Lj} = d_{Lj}(m - d_{Li}).$$

Furthermore, for $k > 0$,

$$\begin{aligned} V_{Lk} = \operatorname{Var} R_{Lk} &= \operatorname{Var}(R_{L,k-1} + r_{Lk}) = V_{L,k-1} + v_{Lk} + 2\operatorname{Cov}(R_{L,k-1}, r_{Lk}) \\ &= V_{L,k-1} + v_{Lk} + 2\sum_{i=0}^{k-1} v_{Lik} = V_{L,k-1} + v_{Lk} + 2\sum_{i=0}^{k-1} d_{Lk}(m - d_{Li}), \end{aligned}$$

that is,

$$V_{Lk} = V_{L,k-1} + v_{Lk} + 2d_{Lk}((k-1)m - D_{L,k-1}),$$

which allows a recursive evaluation of V_{Lk}.

We now want to generalize formulae (3.2) and (3.3). For (3.2) we let

$$P = Er_{L0} + \gamma\sqrt{\operatorname{Var} r_{L0}} = d_{L0} + \gamma\sqrt{v_{L0}}. \tag{4.1}$$

For the kth reinstatement ($k > 0$) it seems natural to let the reinstatement percentage c_k be determined by

$$E\left[\frac{c_k}{m}Pr_{L,k-1}\right] = Er_{Lk} + \gamma\sqrt{\operatorname{Var}\left[r_{Lk} - \frac{c_k}{m}Pr_{L,k-1}\right]}\,; \tag{4.2}$$

the expected premium income should be equal to the expected claims plus a safety loading proportional to the standard deviation of the fluctuating risk, and this fluctuating risk is now equal to $r_{Lk} - \frac{c_k}{m}Pr_{L,k-1}$. For simplicity we introduce the rate on line

$$p_k = \frac{Pc_k}{m}$$

and obtain

$$Ep_k r_{L,k-1} = Er_{Lk} + \gamma\sqrt{\operatorname{Var}(r_{Lk} - p_k r_{L,k-1})},$$

that is,

$$p_k d_{L,k-1} - d_{Lk} = \gamma\sqrt{v_{Lk} + p_k^2 v_{L,k-1} - 2p_k w_{Lk}}\,. \tag{4.3}$$

A problem with the present variant of the standard deviation principle is that (4.3) does not necessarily have a real-valued solution. Let us introduce the function

$$h(x) = d_{L,k-1}x - d_{Lk} - \gamma\sqrt{v_{Lk} + v_{L,k-1}x^2 - 2w_{Lk}x}\,.$$

As $h(d_{Lk}/d_{L,k-1}) < 0$, we see that if the equation $h(x) = 0$ has no real-valued solution, then $h(x) < 0$ for all x. To analyse under what conditions this equation has a solution greater than the pure rate on line $d_{Lk}/d_{L,k-1}$, we could discuss the derivative of h . However, we shall take a less messy and less complete approach. As

$$\lim_{x\uparrow\infty}\frac{h(x)}{x} = d_{L,k-1} - \gamma\sqrt{v_{L,k-1}}\,,$$

we see that if

$$\gamma < d_{L,k-1}/\sqrt{v_{L,k-1}}\,,$$

then $h(x) = 0$ will have a solution greater than the pure rate on line. Thus the equation has an acceptable solution at least for sufficiently small loading factor γ.

By taking the square of (4.3) we obtain

$$(p_k d_{L,k-1} - d_{Lk})^2 = \gamma^2(v_{Lk} + p_k^2 v_{L,k-1} - 2p_k w_{Lk}),$$

which we rearrange as

$$(d_{L,k-1}^2 - \gamma^2 v_{L,k-1})p_k^2 - 2(d_{L,k-1}d_{Lk} - \gamma^2 w_{Lk})p_k + d_{Lk}^2 - \gamma^2 v_{Lk} = 0. \qquad (4.4)$$

This is a quadratic equation with two solutions. As the smaller one is the solution of

$$p_k d_{L,k-1} - d_{Lk} = -\gamma\sqrt{v_{Lk} + p_k^2 v_{L,k-1} - 2p_k w_{Lk}}\,,$$

we are left with

$$p_k = \max\left\{\frac{d_{L,k-1}d_{Lk} - \gamma^2 w_{Lk} \pm \sqrt{(d_{L,k-1}d_{Lk} - \gamma^2 w_{Lk})^2 - (d_{L,k-1}^2 - \gamma^2 v_{L,k-1})(d_{Lk}^2 - \gamma^2 v_{Lk})}}{d_{L,k-1}^2 - \gamma^2 v_{L,k-1}}\right\}. \qquad (4.5)$$

Let us now generalize the premium scheme described by formula (3.4) to loaded premiums. Analogous to (4.2), we let the initial premium P be determined by

$$ET = ER_{LK} + \gamma\sqrt{\mathrm{Var}(R_{LK} - T)}$$

with T given by (2.3), that is,

$$E\left(P\left(1 + \frac{1}{m}\sum_{k=1}^{K} c_k r_{L,k-1}\right)\right) = ER_{LK} + \gamma\sqrt{\mathrm{Var}\left[R_{LK} - \frac{P}{m}\sum_{k=1}^{K} c_k r_{L,k-1}\right]}\,. \quad (4.6)$$

Introduction of the rate on line $p = P/m$ gives

$$E\left(p\left(m + \sum_{k=1}^{K} c_k r_{L,k-1}\right)\right) = ER_{LK} + \gamma\sqrt{\mathrm{Var}(R_{LK} - p\sum_{k=1}^{K} c_k r_{L,k-1})}\,,$$

that is,

$$pA - D_{LK} = \gamma\sqrt{V_{LK} + p^2 B - 2pC} \quad (4.7)$$

with

$$A = m + \sum_{k=1}^{K} c_k d_{L,k-1}$$

$$B = \mathrm{Var}\left(\sum_{k=1}^{K} c_k r_{L,k-1}\right) = \sum_{i=1}^{K}\sum_{j=1}^{K} c_i c_j v_{L,i-1,j-1}$$

$$C = \mathrm{Cov}\left(\sum_{k=1}^{K} c_k r_{L,k-1}, R_{LK}\right) = \sum_{k=1}^{K}\sum_{i=0}^{K} c_k v_{L,k-1,i}.$$

Analogously to our previous discussion, we see that (4.7) has an acceptable solution at least if $\gamma < A/\sqrt{B}$; numerical studies indicate that it has an acceptable solution in most cases of practical interest.
Taking the square of (4.7) gives

$$(pA - D_{LK})^2 = \gamma^2(V_{LK} + p^2 B - 2pC),$$

which we rearrange as

$$(A^2 - \gamma^2 B)p^2 - 2(AD_{LK} - \gamma^2 C)p + D_{LK}^2 - \gamma^2 V_{LK} = 0.$$

This is a quadratic equation with two solutions, but like with equations (4.4), the smaller one drops out, and we are left with

$$p = \max\left\{\frac{AD_{LK} - \gamma^2 C \pm \sqrt{(AD_{LK} - \gamma^2 C)^2 - (A^2 - \gamma^2 B)(D_{LK}^2 - \gamma^2 V_{LK})}}{A^2 - \gamma^2 B}\right\}. \tag{4.8}$$

Let us look at the special case when all reinstatements are at $100c$ %. Then

$$\begin{aligned}
A &= m + cD_{L,K-1} \\
B &= \mathrm{Var}(cR_{L,K-1}) = c^2 V_{L,K-1} \\
C &= \mathrm{Cov}(cR_{L,K-1}, R_{LK}) \\
&= c\{\mathrm{Var}\, R_{L,K-1} + \mathrm{Cov}(R_{L,K-1}, r_{L,k})\} \\
&= c(V_{L,K-1} + d_{Lk}((K-1)m - D_{L,K-1})).
\end{aligned}$$

By letting K approach infinity like in Section 3, we obtain

$$A = m + cD_{L\infty} \quad B = c^2 V_{L\infty} \quad C = cV_{L\infty},$$

where we have introduced

$$V_{L\infty} = \mathrm{Var}\, R_{L\infty} = \overline{\overline{G}}(L) - D_{L\infty}^2 .$$

In particular, $V_{0\infty} = \mathrm{Var}\, X$.

In the special case when all reinstatements are free, (4.6) reduces to

$$P = ER_{LK} + \gamma\sqrt{\mathrm{Var}\, R_{LK}} = D_{LK} + \gamma\sqrt{V_{LK}},$$

which is a traditional application of the standard deviation principle.

There is an inconsistency between the approach for determination of P and the p_k's by (4.1) and (4.5) and the one for determination of p by (4.8). For consistency one should have

$$E\left(P + \sum_{k=1}^{K} p_k r_{L,k-1}\right) = ER_{LK} + \gamma\sqrt{\mathrm{Var}\left(R_{LK} - \sum_{k=1}^{K} p_k r_{L,k-1}\right)}.$$

Unfortunately, this equality is not in general satisfied as

$$\sqrt{\operatorname{Var}\left(r_{L0}+\sum_{k=1}^{K}(r_{Lk}-p_k r_{L,k-1})\right)} \neq \sqrt{\operatorname{Var} r_{L0}}+\sum_{k=1}^{K}\sqrt{\operatorname{Var}(r_{Lk}-p_k r_{L,k-1})}.$$

5 Compound distributions

We now assume that the Y_i's are independent and identically distributed with common distribution F_0 and independent of N. We introduce

$$q_n = Pr(N = n). \quad (n = 0, 1, 2, \ldots)$$

Let F denote the cumulative distribution of the Z_i's given by (2.2). Then

$$F(z) = \begin{cases} F_0(l+z) & (0 \le z < m) \\ 1. & (z \ge m) \end{cases} \tag{5.1}$$

The distribution G is now a compound distribution given by

$$G = \sum_{n=0}^{\infty} q_n F^{n^*}$$

with F^{n^*} denoting the n-fold convolution of F. There exists a vast literature on exact and approximate evaluation of compound distributions.
For the moment we assume that the Y_i's are integer-valued, and that l and m are integers. Let

$$f(z) = Pr(Z = z) \qquad (z = 0, 1, \ldots, m)$$
$$g(x) = Pr(X = x) \qquad (x = 0, 1, \ldots)$$

(we drop indices when they do not contain any information). Then, by (5.1)

$$f(0) = F_0(l)$$
$$f(z) = F_0(l+z) - F_0(l+z-1) \qquad (z = 1, 2, \ldots, m-1)$$
$$f(m) = 1 - F_0(l+m-1).$$

We introduce the tail $G^c = 1 - G$ of G. From the values of g we can evaluate G^c, $\overline{G}$, and $\overline{\overline{G}}$ recursively by

$$\begin{aligned}
G^c(-1) &= 1 \\
\overline{G}(0) &= EX = ENEZ \\
\overline{\overline{G}}\,(0) &= EX^2 = EN \operatorname{Var} Z + EN^2E^2Z. \\
G^c(x) &= G^c(x-1) - g(x) \\
\overline{G}(x) &= \overline{G}(x-1) - G^c(x-1) \\
\overline{\overline{G}}\,(x) &= \overline{\overline{G}}\,(x-1) - 2\overline{G}(x-1) + G^c(x-1).
\end{aligned}$$

We assume that q_n satisfies the recursion

$$q_n = \left[a + \frac{b}{n}\right] q_{n-1}\,. \quad (n = 1, 2, \dots)$$

Theorem 1 in *Sundt/Jewell* (1981) gives that the non-degenerate distributions satisfying this recursion are the Poisson, the negative binomial, and the binomial distributions, and from formula (6.3) in that paper we see that g can be evaluated recursively by

$$g(0) = \begin{cases} e^{-b(1-f(0))} & (a = 0) \\ \left[\dfrac{1 - af(0)}{1-a}\right]^{-\frac{a+b}{a}} & (a \neq 0) \end{cases} \tag{5.2}$$

$$g(x) = \frac{1}{1 - af(0)} \sum_{z=0}^{x} \left[a + b\frac{z}{x}\right] f(z) g(x-z). \quad (x = 1, 2, \dots)$$

The assumption that the Y_i's are integer-valued, is more general than it may seem at first glance as we have not made any restriction to a particular monetary unit. Thus the assumption really means that the Y_i's are distributed on the set $\{0, h, 2h, \dots\}$ for some positive number h. Such a distribution is called *arithmetic,* and the smallest h for which the (Y_i/h)'s are integer-valued, is called the *span* of the distribution.

If the distribution F is not arithmetic, we can approximate it with an arithmetic distribution. Such approximations can be useful even if F actually is arithmetic. Then we approximate it to obtain a larger span and less time-consuming computations. For numerical evaluation of *xl* premiums we have applied the *method of mass dispersal.* We approximate F by a distribution on the $t+1$ points $ih\,(i = 0, 1, \ldots, t)$ with $h = m/t$. For $i = 1, \ldots, t$ we distribute the probability mass $F(ih) - F((i-1)h)$ of the interval $((i-1)h, ih]$ at the two end-points $(i-1)h$ and ih such that the mean is preserved. For the interval $[0, h]$ we proceed in the same way, but include the left end-point. As the mean is preserved for each of the discretisation intervals, the approximation also preserves the total mean of F. Furthermore, it can be shown that the method of mass dispersal gives an upper bound for the stop-loss transform $\overline{G}$. For more details on this method we refer to *Gerber* (1982) and *Panjer/Lutek* (1983), who also discuss other methods of arithmetisation.

6 Example

We have implemented some of the algorithms described in this paper in Mathematica under the assumptions of Section 5 with the additional assumptions that N is Poisson distributed with parameter λ and the Y_i's are Pareto distributed with parameters l and α, that is,

$$q_n = \frac{\lambda^n}{n!} e^{-\lambda} \qquad (n = 0, 1, 2, \ldots, \lambda > 0) \tag{6.1}$$

$$F_0(y) = 1 - (l/y)^{\alpha}. \qquad (y \geq l > 0;\ \alpha > 0) \tag{6.2}$$

From (5.1) and (6.2) we obtain

$$F(z) = \begin{cases} 1 - \left[\dfrac{l}{l+z}\right]^{\alpha} & (0 \leq z < m) \\ 1, & (z \geq m) \end{cases}$$

which we arithmetise as described in Section 5. Under the assumption (6.1), (5.2) reduces to

$$g(0) = e^{-\lambda(1-f(0))}$$

$$g(x) = \frac{\lambda}{x} \sum_{z=1}^{x} z f(z) g(x-z). \qquad (x = 1, 2, \ldots)$$

For $l = m = 100$, $\alpha = 1.2$, and $\lambda = 0.5$ we have calculated pure premiums in Table 1 and loaded premiums with $\gamma = 0.2$ in Table 2 for different aggregate deductibles and reinstatement arrangements with $t = 50$; within each arrangement we have the same reinstatement percentage c for all reinstatements. We also did the same calculations with $t = 10$, and they were remarkably accurate. However, other calculations indicate that the accuracy depends very much on the parameter values. Unfortunately we have not yet been able to deduce upper bounds for the inaccuracy.

	$K =$	0	1	1	2	2	∞	∞
L	$c =$		free	100 %	free	100 %	free	100 %
0		27.85	31.94	24.98	32.33	24.51	32.36	24.45
100		4.088	4.485	4.309	4.514	4.319	4.515	4.320
200		0.3963	0.4247	0.4230	0.4264	0.4245	0.4263	0.4246

Table 1. Pure premiums.

	$K =$	0	1	1	2	2	∞	∞
L	$c =$		free	100 %	free	100 %	free	100 %
0		36.11	42.15	31.10	42.87	30.17	42.93	30.04
100		7.635	8.583	7.983	8.677	7.990	8.682	7.990
200		1.484	1.644	1.621	1.659	1.631	1.659	1.633

Table 2. Loaded premiums.

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Summary

In the present paper we discuss calculation of reinsurance premiums with reinstatements. We consider both pure premiums and premiums loaded by the standard deviation principle. With paid reinstatements the premium income can be considered as random, and therefore we cannot immediately apply the standard deviation principle in the usual way. We discuss premium calculation when the aggregate claim is generated by a compound distribution, and give a numerical example.

Zusammenfassung

In der vorliegenden Arbeit wird die Berechnung von Prämien für Rückversicherungen mit Wiederauffüllung studiert. Es werden sowohl Nettoprämien als auch Bruttoprämien unter Zugrundelegung des Prinzips der Standardabweichung diskutiert. Bei einer vom Zedenten bezahlten Wiederauffüllung können die Prämieneinnahmen als zufallsbedingt angesehen werden; demzufolge kann das Standardabweichungsprinzip nicht in seiner gewöhnlichen Form verwendet werden. Der Autor studiert Prämienberechnungen für den Fall, wo die aufgelaufenen Schäden einer zusammengesetzten Verteilung folgen, und illustriert dies an einem numerischen Beispiel.

Résumé

Le présent article aborde la question du calcul des primes de formes de réassurance avec reconstitution. L'auteur considère tout à la fois les primes pures et les primes chargées selon le principe de l'écart-type. Dans le cas de reconstitutions payées, l'encaisse de prime peut être considérée aléatoire, ce qui ne permet par d'appliquer immédiatement le principe de l'écart-type sous sa forme usuelle. Le calcul des primes est abordé dans le cas de charges annuelles de sinistres générées par une distribution composée. Un exemple numérique termine l'exposé.