Certain extensions of the Chain-Ladder technique

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Certain Extensions of the Chain-Ladder Technique

1. Introduction

During the past 20 years a lot has been written on how to calculate adequate loss reserves in nonlife insurance. Certain survey books were published (see e.g. *Van Eeghen* (1980), *Taylor* (1984), *Institute of Actuaries* (1989)) and the topic was included in standard actuarial textbooks (see e.g. *Sundt* (1983), *Kremer* (1985)). Nevertheless the field does not seem to be totally complete, further new methods and refined techniques can be expected. Newer approaches have to be developed further and prepared for application (see e.g. the approach of *Kremer* (1989)). In practice certain methods are applied already with great success showing the high value of newer actuarial research. In 1984 the author published a paper discussing an important type of loss reserving techniques, the so called autoregressive models. The given methods are generalisations of the classical, simple chain ladder technique and consequently are in practical use. The underlying models are fairly handy since they are affine in type. The question arises if one can give also certain nonaffine modifications of the models that are handy and good enough for successful practical application. This question can be answered positively as is shown below.

2. The model

Let $X_\triangle = \left(X_{ij},\ j = 1, \ldots, n-i+1,\ i = 1, \ldots, n\right)$ be the run-off triangle of a risk or collective of risks. This means that the X_{ij} is a random variable on the probability space (Ω, A, P) describing the total claims amount of accident year no. i with respect to its development year no. j. One assumes that one has:

(1) functions f_{j,a_j} on $(0, \infty)$ depending on the development year j and a parameter vector $a_j = (a_{j1}, \ldots, a_{jp_j})$

(2) random (error) variables R_{ij} defined on $(\Omega, \triangle, P)$ such that one has the model:

$$X_{ij} = f_{j,a_j}(X_{i,j-1}) \star_j R_{ij}\,, \tag{2.1}$$

where "$\star_j$" is an operator on $IR \times IR$ into IR, allowed to depend on the development year. Suppose that:

(A.1) the $R_{ij},\ i = 1, \ldots, n,\ j = 1, \ldots, n$ are stochastically independent and square integrable,

(A.2) the $X_{i1},\ i = 1, \ldots, n$ are stochastically independent and square integrable,

(A.3) the $R_{ij},\ i = 1, \ldots, n,\ j = 1, \ldots, n$ are stochastically independent of the $X_{i1},\ i = 1, \ldots, n$.

These three conditions imply that:

$$\text{the } X_{il},\ l \leq j-1 \text{ are stochastically independent of the } R_{ij}\,. \tag{2.2}$$

The two relevant choices for the operator "$\star_j$" are clearly:

$$\text{"}\star_j\text{"} = \text{"}+\text{"} \tag{2.3}$$

and:

$$\text{"}\star_j\text{"} = \text{"}\cdot\text{"} \tag{2.4}$$

Models of type (2.1) with the choice (2.3) are intensively explored in a paper of *Jones* (1978).

3. Optimal predictions

In loss reserving the $X_{ij},\ j \geq n-i+2$ are unknown. One has to predict them from the run-off-triangle $X_\triangle$ in an optimal way. For giving an adequate prediction advice let us take two additional assumptions:

(A.4) with the expectations $r_{ij} = E(R_{ij})$ one has:

$$f_{j,a_j}(X_{i,j-1}) \star_j r_{ij} = f_{j,a_j}(X_{i,j-1})$$

(A.5) for stochastically independent, square-integrable random variables X, Y and a random vector Z (all defined on $(\Omega, \triangle, P)$) one has the rule:

$$E(X \star_j Y \mid Z) = E(X \mid Z) \star_j E(Y \mid Z)$$

One knows (see e.g. *Kremer* (1984)) that the optimal (=adequate) prediction $\widehat{X}_{ij}$ of X_{ij}, $j \geq n-i+2$ is just a conditional expectation:

$$\widehat{X}_{ij} = E(X_{ij} \mid X_{\triangle}) \tag{3.5}$$

Under the given assumptions one has as basic result:

Theorem 1

The prediction (3.5) is nothing else but:

$$\widehat{X}_{ij} = f_{j,\,a_j}(X_{i,\,j-1}), \quad j \geq n-i+2$$

Proof: Because of (A.1)–(A.3), (A.5) $\widehat{X}_{ij}$ is equal to:

$$E(f_{j,\,a_j}(X_{i,\,j-1}) \mid X_{il},\, l \leq n-i+1) \star_j E(R_{ij} \mid X_{il},\, l \leq n-i+1)$$

The first term is equal to $f_{j,\,a_j}(X_{i,\,j-1})$, the second to r_{ij} because of (2.1), (2.2). The statement follows because of (A.4). □

For $j \geq n-i+2$ the $X_{i,\,j-1}$ is not known, one clearly replaces it by its prediction $\widehat{X}_{i,\,j-1}$. This results in the following advice:
Complete the triangle $X_{\triangle}$ to a rectangle by predicting X_{ij}, $j \geq n-i+2$ by $\widehat{X}_{ij}$ with the recursion:

$$\boxed{\begin{aligned} \widehat{X}_{i,n-i+2} &= f_{n-i+2,\,a_{n-i+2}}(X_{i,n-i+1}) \\ \widehat{X}_{i,\,n-i+k} &= f_{n-i+k,\,a_{n-i+k}}(\widehat{X}_{i,n-i+k-1}) \quad k \geq 3 \end{aligned}} \tag{3.6}$$

When one has given the functions $f_{j,\,a_j}$ and knows the value of the parameter vector a_j, the completion of the triangle $X_{\triangle}$ to a rectangle can be carried through. Nevertheless in practice one does not know the (correct) underlying value of the parameter-vector a_j, its value has to be estimated.

4. Parameter estimation

For estimating the unknown value of the parameter-vector one can take the known data of the run-off-triangle as in *Kremer* (1984). In addition to the assumptions (A.1)–(A.4) it is assumed that for each j there exists a measurable mapping T_j

on IR into IR such that:

(A.6) $T_j(f_{j,\,a_j}(X_{i,j-1}) \star_j R_{ij}) = T_j(f_{j,a_j}(X_{i,\,j-1})) + T_j(R_{ij})$
(A.7) $E(T_j(R_{ij})) = 0$
(A.8) $\operatorname{Var}(T_j(R_{ij})) = s_j / V_{ij}$
where $s_j \geq 0$ and V_{ij} are for $i = 1, \ldots, n, \;\; j = 1, \ldots, n$
positive volume measures.

The assumption (A.6) implies with (2.1) that:

$$T_j(X_{ij}) = T_j(f_{j,\,a_j}(X_{i,\,j-1})) + T_j(R_{ij}) \tag{4.1}$$

Because of (A.1), (A.7), (A.8) and (4.1) one is willing to calculate the desired estimator $\widehat{a}_j$ for a_j with the following method of least squares:

$$\sum_{i=1}^{n-j+1} V_{ij} \cdot (T_j(X_{ij}) - T_j(f_{j,\,a_j}(X_{i,\,j-1})))^2 = \min_{a_j}$$

With the common method of the differential calculus one obtains then:

Theorem 2

The least squares estimator $\widehat{a}_j$ satisfies the normal equations:

$$\begin{aligned} &\sum_{i=1}^{n-j+1} V_{ij} \cdot T_j(X_{ij}) \cdot d_{j,\widehat{a}_j}^{(k)}(X_{i,\,j-1}) \\ &\quad = \sum_{i=1}^{n-j+1} V_{ij} \cdot T_j(f_{j,\,\widehat{a}_j}(X_{i,\,j-1})) \cdot d_{j,\widehat{a}_j}^{(k)}(X_{i,j-1}) \quad \text{for } k = 1, \ldots, p_j, \end{aligned} \tag{4.2}$$

with the derivatives:

$$d_{j,\,a_j}^{(k)} = \left(\frac{d\, T_j(f_{j,\,a_j})}{d\, a_{jk}} \right). \qquad \square$$

In the special case that:

$$s_j = 0 \tag{4.3}$$

one has because of (A.7):

$$T_j(R_{ij}) = 0 \quad \text{f.s.}$$

and consequently from (4.1):

$$T_j(X_{ij}) = T_j(f_{j,\,a_j}(X_{i,j-1})), \quad \text{f.s., for all } i = 1, \ldots, n-j+1.$$

If in addition to (4.3) one has:

$$p_j = 1$$

one is willing to take as estimator $\widehat{a}_j = (\widehat{a}_{j1})$ of $a_j = (a_{j1})$ the solution (assumed to exist uniquely) of the equation:

$$\sum_{i=1}^{n-j+1} T_j(X_{ij}) = \sum_{i=1}^{n-j+1} T_j(f_{j,\,\widehat{a}_j}(X_{i,\,j-1})) \tag{4.4}$$

The equation (4.4) is slightly simpler than the corresponding equation (4.2) (with $p_j = 1$), but the estimator of (4.2) is more adequate in case that s_j is larger than zero.
For the *special choices:*

$$\begin{aligned} &(2.3), \quad f_{j,\,a_j}(x) = a_{j1} \cdot x, \quad a_j = (a_{j1}), \quad p_j = 1 \\ &(4.3), \quad T_j(x) = x \quad \text{for all } x \end{aligned}$$

one gets from (4.4) as estimator $\widehat{a}_{j1}$:

$$\widehat{a}_{j1} = \frac{\sum_{i=1}^{n-j+1} X_{ij}}{\sum_{i=1}^{n-j+1} X_{i,\,j-1}} \tag{4.5}$$

which defines with (3.6) the classical so-called *chain-ladder loss-reserving technique.*
Consequently the method (3.6) with the above parameter estimators (of (4.2) or (4.4)) can be regarded as being a certain generalization of the classical chain-ladder technique.

5. Examples

Clearly many examples can be given for the model (2.1) with assumptions (A.1)–(A.8). The most important ones shall be given in the sequel.

Example 1

Take in the above general model the more special choices given according:

$$
\begin{aligned}
&(2.3), \quad f_{j,a_j}(x) = a_{j1} \cdot x, \quad a_j = (a_{j1}), \quad p_j = 1 \\
&r_{ij} = 0 \quad \text{and finally } T_j(x) = x
\end{aligned}
$$

The estimator $\widehat{a}_j = (\widehat{a}_{j1})$ for $a_j = (a_{j1})$ resulting from (4.4) is already given at the end of section 4. From (4.2) one gets the more suitable estimator:

$$
\widehat{a}_{j1} = \frac{\sum_{i=1}^{n-j+1} V_{ij} \cdot X_{ij} \cdot X_{i,\,j-1}}{\sum_{i=1}^{n-j+1} V_{ij} \cdot X_{i,\,j-1}^2} \tag{5.6}
$$

Note that this example was already given in *Kremer* (1984) in the context of autoregressive models of a linear type. △

Example 2

Choose in the above context:

$$
\begin{aligned}
&(2.3), \quad f_{j,a_j} = a_{j1} \cdot x^{1/2}, \quad a_j = (a_{j1}), \quad p_j = 1 \\
&r_{ij} = 0 \quad \text{and finally} \quad T_j(x) = x
\end{aligned}
$$

The estimator $\widehat{a}_j = (\widehat{a}_{j1})$ for $a_j = (a_{j1})$ resulting from (4.4) is just:

$$
\widehat{a}_{j1} = \frac{\sum_{i=1}^{n-j+1} X_{ij}}{\sum_{i=1}^{n-j+1} X_{i,\,j-1}^{1/2}} \tag{5.7}
$$

and the estimator resulting from (4.2):

$$
\widehat{a}_{j1} = \frac{\sum_{i=1}^{n-j+1} V_{ij} \cdot X_{ij} \cdot X_{i,j-1}^{1/2}}{\sum_{i=1}^{n-j+1} V_{ij} \cdot X_{i,\,j-1}} \qquad \triangle \tag{5.8}
$$

Example 3

Take now the additional model assumptions:

$$(2.4), \quad f_{j,a_j}(x) = \exp(a_{j1} \cdot x), \quad a_j = (a_{j1}), \quad p_j = 1$$
$$r_{ij} = 1 \quad \text{and finally} \quad T_j(x) = \ln(x).$$

The estimator $\widehat{a}_j = (\widehat{a}_{j1})$ for $a_j = (a_{j1})$ resulting from (4.4) is given by:

$$\widehat{a}_{j1} = \frac{\sum_{i=1}^{n-j+1} \ln(X_{ij})}{\sum_{i=1}^{n-j+1} X_{i,j-1}} \tag{5.9}$$

and the estimator resulting from (4.2) by:

$$\widehat{a}_{j1} = \frac{\sum_{i=1}^{n-j+1} V_{ij} \cdot \ln(X_{ij}) \cdot X_{i,j-1}}{\sum_{i=1}^{n-j+1} V_{ij} \cdot X_{i,j-1}^2} \qquad \triangle \tag{5.10}$$

These were examples with $p_i = 1$. Remember that for positive s_j (what mostly is the case in practice) the estimators (5.6), (5.8), (5.10) are in principal preferable to the estimators (4.5), (5.7), (5.9), since they are expected to be more efficient. Nevertheless for manual computations the estimators (4.5), (5.7), (5.9) may sometimes be preferable since they are slightly simpler and quicker to compute. Now turn to cases with $p_j = 2$. In these cases the equation (4.4) is not relevant any more.

Example 4

Choose now in the context of sections 2–4:

$$(2.3), \quad f_{j,a_j}(x) = a_{j1} \cdot x + a_{j2}, \quad a_j = (a_{j1}, a_{j2})$$
$$p_j = 2, \quad r_{ij} = 0 \quad \text{and finally} \quad T_j(x) = x$$

From the equations (4.2) one gets as parameter estimators $\widehat{a}_{ji}$ for a_{ji} the following:

$$\widehat{a}_{j1} = \left(\sum_{i=1}^{n-j+1} \overline{V}_{ij} \cdot \left(X_{ij} - \overline{X}_{\cdot j}^{(j)} \right) \cdot \left(X_{i,j-1} - \overline{X}_{\cdot j-1}^{(j)} \right) \right)$$
$$\times \left(\sum_{i=1}^{n-j+1} \overline{V}_{ij} \cdot \left(X_{i,j-1} - \overline{X}_{\cdot j-1}^{(j)} \right)^2 \right)^{-1}$$

$$\widehat{a}_{j2} = \sum_{i=1}^{n-j+1} \overline{V}_{ij} \cdot X_{ij} - \widehat{a}_{j1} \cdot \overline{X}_{\cdot j-1}^{(j)}$$

with the conventions:

$$\overline{V}_{ij} = V_{ij} \Big/ \Big(\sum_{l=1}^{n-j+1} V_{lj} \Big) \tag{5.11}$$

$$\overline{X}^{(l)}_{\cdot k} = \sum_{i=1}^{n-l+1} \overline{V}_{il} \cdot X_{ik} \tag{5.12}$$

Obviously for $j = n$ the $\widehat{a}_{j1}$ is not well-defined. For $j = n$ the model is not applicable. Then one will take the model of example 1. Note that also this example is contained in Kremer (1984) as a special case of an affine autoregressive model. $\triangle$

Example 5

Take the following additional conditions:

$$\begin{aligned} &(2.3), \quad f_{j,a_j}(x) = a_{j1} \cdot (x - a_{j2})^{1/2}, \quad a_j = (a_{j1}, a_{j2}), \\ &p_j = 2, \quad r_{ij} = 0 \quad \text{and finally} \quad T_j(x) = x \end{aligned}$$

The equations (4.2) give as estimator $\widehat{a}_{j1}$ of a_{j1} the following:

$$\widehat{a}_{j1} = \frac{\sum_{i=1}^{n-j+1} \overline{V}_{ij} \cdot X_{ij} \cdot (X_{i,j-1} - \widehat{a}_{j2})^{1/2}}{\sum_{i=1}^{n-j+1} \overline{V}_{ij} \cdot (X_{i,j-1} - \widehat{a}_{j2})}$$

where the estimator $\widehat{a}_{j2}$ of a_{j2} is the zero place of the equation:

$$\begin{aligned} &\sum_{i=1}^{n-j+1} \overline{V}_{ij} \cdot X_{ij} \cdot (X_{i,j-1} - \widehat{a}_{j2})^{-1/2} \cdot \sum_{i=1}^{n-j+1} \overline{V}_{ij} \cdot (X_{i,j-1} - \widehat{a}_{j2}) \\ &\quad = \sum_{i=1}^{n-j+1} \overline{V}_{ij} \cdot X_{ij} \cdot (X_{i,j-1} - \widehat{a}_{j2})^{1/2} \end{aligned} \tag{5.13}$$

and the $\overline{V}_{ij}$ is defined by (5.11).
For $j = n$ (5.13) does not give a solution since both sides of the equation are equal. Then one will take the model of example 1 or 2 instead. $\triangle$

Example 6

Choose in this final case:

$$\begin{aligned} &(2.4), \quad f_{j,a_j}(x) = a_{j1} \cdot \exp(a_{j2} \cdot x), \quad a_j = (a_{j1}, a_{j2}), \\ &p_j = 2, \quad r_{ij} = 1 \quad \text{and finally} \quad T_j(x) = \ln(x) \end{aligned}$$

In this case it is more practical to introduce a transformed parameter $a_{j1}^\star = \ln(a_{j1})$ and then to give a parameter estimator $\widehat{a}_j^\star = (\widehat{a}_{j1}^\star, \widehat{a}_{j2}^\star)$ for $a_j^\star = (a_{j1}^\star, a_{j2})$. The adequate estimator $\widehat{a}_j = (\widehat{a}_{j1}, \widehat{a}_{j2})$ for a_j is then given simply by

$$\widehat{a}_{j1} = \exp(\widehat{a}_{j1}^\star), \qquad \widehat{a}_{j2} = \widehat{a}_{j2}^\star$$

The slightly modified equations (4.2) give as estimator $\widehat{a}_{ji}^\star$ for $a_{ji}^\star$ the following:

$$\widehat{a}_{j1}^\star = \sum_{i=1}^{n-j+1} \overline{V}_{ij} \cdot \ln(X_{ij}) - \widehat{a}_{j2}^\star \cdot \overline{X}_{\cdot j-1}^{(j)}$$

$$\widehat{a}_{j2}^\star = \Bigg(\sum_{i=1}^{n-j+1} \overline{V}_{ij}.(\ln(X_{ij}) - \sum_{i=1}^{n-j+1} \overline{V}_{ij} \cdot \ln(X_{ij}))$$

$$\times \left(X_{i,j-1} - \overline{X}_{\cdot j-1}^{(j)}\right)\Bigg) \cdot \left(\sum_{i=1}^{n-j+1} \overline{V}_{ij} \cdot \left(X_{i,j-1} - X_{\cdot j-1}^{(j)}\right)^2\right)^{-1}$$

with again the conventions (5.11), (5.12). Obviously for $j = n$ the $\widehat{a}_{j2}^\star$ is not well defined. Then one will take the model of example 1 or 3. △

It is the author's conviction that models with p_j larger than 2 are of limited practical usefulness because parameter estimation becomes unwieldy.

6. Practical procedure

For practical calculations the author proposes the following combination of the models of part 5:
For $j = n$ take the model of example 1. For each previous j adapt each model of the examples 4–6. Then calculate for each adapted model the mean squared prediction error:

$$QS = \sum_{i=1}^{n-j+1} \left(\frac{V_{ij}}{V_{\cdot j}}\right) \cdot (X_{ij} - f_{j,\widehat{a}_j}(X_{i,j-1}))^2 \quad \text{with } V_{\cdot j} = \sum_{i=1}^{n-j+1} V_{ij}$$

For the prediction from development year no. $j-1$ to development year no. j one will take that model with the smallest QS-value then.
Sometimes the models of examples 4–6 are inadequate. One can find this out with the values of the parameter estimators. The model of example 4 can be judged as

being inadequate when $\widehat{a}_{j1}$ is not positive. The model of example 5 can be judged as being inadequate when the $\widehat{a}_{j2}$ is very small (negative!). Finally the model of example 5 can be judged as being inadequate when $\widehat{a}_{j2}$ is not positive. Instead of those models one can take the model of example 1. One expects to get slightly improved predictions in case one applies the above proposed procedure instead of carrying through simply the chain-ladder loss reserving advice.

7. Numerical example

For demonstration of the procedure proposed in section 6 a numerical example is given below. It is assumed that:

$$V_{ij} = 1, \quad \text{for all } i \text{ and } j$$

The run-off triangle of the total claim amounts $(X_{ij},\ j = 1, \ldots, n - i + 1,\ i = 1, \ldots, n)$ is given in *Table 1*. Carried through are the chain-ladder method and the procedure proposed in section 6. For each development year the calculations are given. *Table 2* contains the predictions with the chain-ladder technique, *Table 3* the predictions with the procedure of section 6.

Table 1: The run-off triangle

26.50	50.05	62.54	76.57	87.50	93.05
31.28	48.98	67.39	79.14	85.43	
26.47	47.53	64.51	74.47		
37.77	49.39	62.65			
27.06	45.49				
29.58					

First take the chain-ladder technique given at the end of section 4. The estimator of (4.5) is given by:

	$j = 2$	3	4	5	6
$\widehat{a}_{j1}$	1.6195	1.3120	1.1838	1.1106	1.063

and the QS-values

	$j = 2$	3	4	5	6
QS	43.3902	7.2005	3.4769	6.0620	0.0000

One gets the predicted lower light triangle:

Table 2: Predictions with the chain-ladder method

				90.85
			82.71	87.95
		74.16	82.37	87.59
	59.68	70.65	78.47	83.44
47.91	62.85	74.40	82.63	87.88

Now take the procedure of section 6.

1) Development year $j = 2$:

For the model of example 4 one gets the estimators:

$$\widehat{a}_{21} = 0.1490, \quad \widehat{a}_{22} = 43.8458$$

and the QS-value: $QS = 2.2176$.
For the model of example 5 one gets the estimators:

$$\widehat{a}_{21} = 3.8057, \quad \widehat{a}_{22} = -131.26$$

and the QS-values: $QS = 2.2176$.
Finally for the model of example 6 one gets the estimators:

$$\widehat{a}_{21} = 43.9471, \quad \widehat{a}_{22} = 0.00314$$

and the QS-value: $QS = 2.2185$.
Obviously the models of examples 4 and 5 are equally good. One will prefer for predicting the development year $j = 2$ the simpler model of example 4.

2) Development year $j = 3$:

For the model of example 4 one gets the estimators:

$$\widehat{a}_{31} = -0.8384, \quad \widehat{a}_{32} = 105.3437$$

and the QS-value: $QS = 3.2521$.
For the model of example 5 one gets the estimators:

$$\widehat{a}_{31} = 0.2032, \quad \widehat{a}_{32} = -99\,999.5$$

and the QS-value: $QS = 3.8528$.
Finally for the model of example 6 one gets the estimator:

$$\widehat{a}_{31} = 122.6007, \quad \widehat{a}_{32} = -1.3192$$

and the QS-value: $QS = 3.2620$.
According to what was said in section 6 all models can be judged to be inadequate. One will take the model of example 1 instead. From (4.6) one gets the parameter estimator:

$$\widehat{a}_{31} = 1.3112$$

with the QS-value: $QS = 7.1991$.

3) Development year $j = 4$:

For the model of example 4 one gets the estimators:

$$\widehat{a}_{41} = 0.6100, \quad \widehat{a}_{42} = 37.1880$$

and the QS-value: $QS = 2.1710$.
For the model of example 5 one gets the estimators:

$$\widehat{a}_{41} = 9.6981, \quad \widehat{a}_{42} = 2.2052$$

and the QS-value: $QS = 2.1902$.
Finally for the model of example 6 one gets the estimators:

$$\widehat{a}_{41} = 46.0933, \quad \widehat{a}_{42} = 0.00785$$

and the QS-value: $QS = 2.1528$.
Obviously for predicting the development year $j = 4$ the model of example 6 is the best one.

4) Development year $j = 5$:

For the model of example 4 one gets the estimators:

$$\widehat{a}_{51} = -0.8054, \quad \widehat{a}_{52} = 149.1731$$

and the QS-value: $QS = 0.0000$.

For the model of example 5 one gets the estimators:

$$\widehat{a}_{51} = 0.3266, \quad \widehat{a}_{52} = -70\,000$$

and the QS-value: $QS = 1.07282$.
Finally for the model of example 6 one gets the estimators:

$$\widehat{a}_{51} = 178.564, \quad \widehat{a}_{52} = -0.00931$$

and the QS-value: $QS = 0.0000$.
Obviously all models can be judged as being inadequate. One will take the model of example 1. One gets from (4.6) as estimator:

$$\widehat{a}_{51} = 1.1101$$

with the QS-value: $QS = 6.0603$.

5) Development year $j = 6$:

One has to take the model 1 with the estimator (4.5) of the chain-ladder method, given already above.

Combining the steps 1–5 one arrives at the following predicted triangle:

Table 3: Predictions with the procedure of part 6

				90.85
			82.67	87.88
		75.41	83.71	88.98
	59.65	73.65	81.75	86.90
48.25	63.27	75.78	84.12	89.42

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Summary

The problem of calculating adequate loss reserves is reconsidered. Certain simpler, not necessary affine autoregressive models are introduced and with them optimal loss-development predicting advices derived. The resulting methods are in some sense extensions of the classical chain-ladder loss-reserving technique.

Zusammenfassung

Das Problem der Bestimmung adäquater Schadenrückstellungen wird betrachtet. Bestimmte einfachere, nicht notwendigerweise affine autoregressive Modelle werden eingeführt. Mit ihnen werden optimale Prognosen zur Entwicklung des Schadenverlaufs abgeleitet. Die resultierenden Methoden sind in gewissem Sinn Erweiterungen der klassischen Chain-ladder-Schadenreservierungstechnik.

Résumé

Le problème du calcul de réserves de sinistres adéquates est présenté. Certains modèles plus simples, pas nécessairement affins autorégressifs sont introduits, avec lesquels des prévisions optimales de l'evolution des sinistres sont dérivées. Les méthodes résultantes sont en quelque sorte des extensions de la technique classique ≪chain-ladder≫ du provisionnement des sinistres.