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Corrected Diffusion Approximations for a Risk Process with the Possibility of Borrowing and Investment

1 Introduction

In classical risk theory in general no explicit formulae for the probability of ruin can be obtained. Therefore approximations are called for. For the Cramér-Lundberg model, some of the most important approximation methods can be found in [1]. A useful method also described therein is the called *diffusion approximation*. The idea is to consider an appropriate diffusion process instead of the risk process and to use the corresponding ruin probabilities as an approximation.
As far as I know the first treatment of diffusion approximations in risk theory goes back to H. Hadwiger [11] in 1940. The modern approach, based on weak convergence, is due to Iglehart [12]. The idea is to let increase the number of claims in a unit time interval and to make the claim sizes smaller in such a way that the risk process converges weakly to a diffusion. Another interpretation of the convergence can be found in [9].
The easiest way to do this is to use a sequence of risk processes having the same drift and the same auto-covariance function as the approximated risk process. But comparing the approximated values it turns out, that this approximation satisfies only in cases where the safety loading is very small. For a comparison of diffusion approximations and exact values see for instance [9].
The main problem with the approximation seems to be the overshoot, that means the difference between the ruin level and the surplus of the process at the ruin time, which is smoothed by a diffusion. It is due to this effect that corrected diffusion approximations are constructed (see for instance [1] or [18]).
The problem how these classical diffusion approximations can be used to construct approximations for more general models was recently solved by Schmidli [16], [15]. He compared the approximation results with exact values for a risk model also including investment and borrowing. The purpose of the present paper is to show, how the approximation results can be improved by using corrected diffusion approximations.

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The considered approximations are based on weak convergence. Denote by D the space of all càdlàg (i.e. right-continuous with left-hand limits) functions on $[0, \infty)$ endowed with the Skorohod topology (see for instance [6] and references therein). In the sequel all stochastic processes are assumed to be in D. Recall the definition of weak convergence.

Definition 1. *A sequence $(X^{(n)} : n \in \mathbb{N})$ of stochastic processes is said to converge weakly to a stochastic process X if for every bounded continuous functional f it follows that*

$$\lim_{n\to\infty} E[f(X^{(n)})] = E[f(X)]\,.$$

In this case we write $X^{(n)} \Longrightarrow X$.

For further background on weak convergence see also [2], [6], [13] and [14].

2 Risk Processes in an Economic Environment

Let us start defining the basic model. Let in the sequel $(\Omega, \mathfrak{F}, P)$ be a large enough probability space, in particular carrying the following independent objects

- a Poisson process $(N_t : t \geq 0)$ with rate $\lambda > 0$;
- a sequence $(Y_i : i \in \mathbb{N})$ of i.i.d. random variables, having the distribution function G, with $G(0) = 0$, mean μ and finite variance σ^2.

Set

$$S_t := \sum_{i=1}^{N_t} Y_i$$

the accumulated claim process and define the classical risk process $(C_t : t \geq 0)$ by

$$C_t := u + ct - S_t$$

where $c > 0$ is the premium income rate and u denotes the initial capital. Denote by $\varrho := (c - \lambda\mu)/(\lambda\mu)$ the safety loading. We call above model also Cramér-Lundberg model. A comprehensive discussion of this model can be found for instance in [10].

Let $(C_t^{(n)} : n \in \mathbb{N})$ be a sequence of classical risk processes. Denote the corresponding Poisson processes by $(N_t^{(n)})$, their intensities by $\lambda^{(n)}$, the claims by $Y_i^{(n)}$, their distribution function by $G^{(n)}$, the initial capital by u, the premium rates by $c^{(n)}$ and the accumulated claim processes by $(S_t^{(n)})$, i.e. $C_t^{(n)} = u + c^{(n)}t - S_t^{(n)}$. Set

$$\mu^{(n)} := \int_0^\infty x\, dG^{(n)}(x)\,, \quad \sigma^{(n)^2} := \int_0^\infty (x - \mu^{(n)})^2\, dG^{(n)}(x)$$

where we assume $\sigma^{(n)^2} < \infty$. For each of the processes we define the safety loading $\varrho^{(n)} := (c^{(n)} - \lambda^{(n)}\mu^{(n)})/(\lambda^{(n)}\mu^{(n)})$. Denote by $(Q_t : t \geq 0)$ a (d, η^2)-Brownian motion, i.e. $Q_t = Q_0 + \eta B_t + dt$ where $(B_t : t \geq 0)$ is a standard Wiener process. We assume in the sequel that $C^{(n)} \Longrightarrow Q$. If the sequence is chosen appropriately, i.e. all $C^{(n)}$'s have 'similar properties', then we call Q a diffusion approximation for $C^{(1)}$. For the classical theory of diffusion approximations, similar properties means that $E[C_1^{(n)}] = E[C_1^{(1)}]$ and $\mathrm{Var}\,[C_1^{(n)}] = \mathrm{Var}\,[C_1^{(1)}]$. For more background on diffusion approximations for the classical risk model see [12] or [9].

In the sequel we want to consider a risk model, where the drift depends on the surplus of the process, e.g. by allowing the company to invest or to borrow money. Let

$$\delta(x) := \begin{cases} \beta_1(x - \Delta) & x \geq \Delta, \\ 0 & 0 \leq x < \Delta, \\ \beta_2 x & x < 0\,. \end{cases} \tag{1}$$

We define a sequence $(X^{(n)} : n \in \mathbb{N})$ of risk processes via the stochastic differential equation (SDE)

$$dX_t^{(n)} = \delta(X_t^{(n)})\, dt + dC_t^{(n)}, \quad X_0^{(n)} = C_0^{(n)}\,,$$

and a diffusion Z satisfying the SDE

$$dZ_t = \delta(Z_t)\, dt + dQ_t, \quad Z_0 = Q_0\,.$$

Recall from [4, p. 183] that $X^{(n)}$ as well as Z are well-defined. The parameter β_1 is interpreted as the force of interest for invested money, β_2 as the force of interest for borrowed money and Δ as the amount of money the company retains as a liquid reserve. We define the ruin time $\tau^{(n)}$ as the first epoch, where the payments for interest are larger than the premium income, i.e. $\tau^{(n)} := \inf\{t > 0 : c < -\beta_2 X_t^{(n)}\}$. It is easy to see that

$$\{\tau^{(n)} < \infty\} = \{X_t^{(n)} \to -\infty\} \text{ a.s.}$$

where, also in the sequel, $\to$ denotes the limit for $t \to \infty$. A discussion of the model defined above can be found in [15], [5] or [17].
Recall that we assumed that $C^{(n)} \Longrightarrow Q$. It follows now from [16, Thm.2] that also $X^{(n)} \Longrightarrow Z$ and furthermore that the infinite time ruin probabilities of the processes $X^{(n)}$ converge to the infinite time ruin probability of Z, i.e.

$$\lim_{n\to\infty} P[X_t^{(n)} \to -\infty] = P[Z_t \to -\infty],$$

provided that $\overline{\lim}_{n\to\infty} \lambda^{(n)}(\sigma^{(n)^2} + \mu^{(n)^2}) < \infty$ ([16, Thm.3]).

Remarks.

i) Note that the premium rate c does not appear in (1). It is hidden in $dC_t^{(n)}$ and therefore also present in $dX_t^{(n)}$.

ii) By setting the parameters $\Delta = 0$ and $\beta_1 = \beta_2$ we get the model considered by Gerber [7]. By setting the parameter $\Delta = \infty$ we get as a special case the model considered by Dassios and Embrechts [3].

iii) Notice that in the case $\Delta < \infty$, contrary to the classical case, there is no net profit condition needed to assure $P[\tau^{(n)} < \infty] \neq 1$ (see also [5]).

iv) Note that for $n \to \infty$ the ruin barrier converges to $-\infty$. Therefore we shall consider the event $\{X_t^{(n)} \to -\infty\}$ as our ruin event, which also makes sense for the process (Z_t).

v) The condition $\overline{\lim}_{n\to\infty} \lambda^{(n)}(\sigma^{(n)^2} + \mu^{(n)^2}) < \infty$ is rather weak. Note that $\mathrm{Var}[C_1^{(n)}] = \lambda^{(n)}(\sigma^{(n)^2} + \mu^{(n)^2})$. It would not make sense to consider a sequence where $\mathrm{Var}[C_1^{(n)}]$ is unbounded and to use a diffusion with $\mathrm{Var}[Z_1] = \eta^2$ as an approximation. ■

To end this section we recall from [16] or [8] an explicit expression for the ruin probability of the diffusion Z.

Proposition 1. *Let Z be the diffusion defined above. Then the probability of ultimate ruin is given by*

$$P[Z_t \to -\infty] = \frac{f(\infty) - f(Z_0)}{f(\infty) - f(-\infty)}$$

where f denotes the function $f : \mathbb{R} \to \mathbb{R}$, *with*

$$f(x) = \begin{cases} \frac{\eta}{d}\Big(1 - \exp\Big(-\frac{2d}{\eta^2}\Delta\Big)\Big) + 2\sqrt{\frac{\pi}{\beta_1}}\exp\Big(\frac{d^2}{\eta^2\beta_1} - \frac{2d}{\eta^2}\Delta\Big) & \\ \quad \cdot \Big(\Phi\big(\frac{\sqrt{2\beta_1}}{\eta}\big(x - \Delta + \frac{d}{\beta_1}\big)\big) - \Phi\Big(\frac{d}{\eta}\sqrt{\frac{2}{\beta_1}}\Big)\Big) & \Delta \le x\,, \\ \frac{\eta}{d}\Big(1 - \exp\Big(-\frac{2d}{\eta^2}x\Big)\Big) & 0 \le x < \Delta\,, \\ 2\sqrt{\frac{\pi}{\beta_2}}\exp\Big(\frac{d^2}{\eta_2\beta_2}\Big(\Phi\Big(\frac{\sqrt{2\beta_2}}{\eta}\Big(x + \frac{d}{\beta_2}\Big)\Big) - \Phi\Big(\frac{d}{\eta}\sqrt{\frac{2}{\beta_2}}\Big)\Big) & x < 0\,, \end{cases}$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-y^2/2}\,dy$. □

Remark. If we use the uncorrected diffusion approximation approach the parameters of the diffusion are given by $d = c - \lambda\mu$ and $\eta^2 = \lambda(\sigma^2 + \mu^2)$. For the special case $\beta_1 = \beta_2$ and $\Delta = 0$ we recover the approximation proposed by Gerber [7]. His approach was based on an asymptotic normality property as $\beta_1^{-1}\lambda \to \infty$. ■

A comparison of the approximation and exact results for exponential distributed claims can be found in [16], where the risk process is approximated by an 'uncorrected' diffusion with the same drift and auto-covariance function.

3 Corrected Approximations

Let us first consider a diffusion approximation $C^{(n)} \Longrightarrow Q$ for the classical Cramér-Lundberg model. Assume that the Lundberg coefficient, i.e. the strictly positive solution γ_1 of the equation

$$h(r) := \lambda(\widehat{G}(-r) - 1) - cr = 0$$

exists. Denote by γ_0 is the unique solution of the equation $h'(r) = 0$ and by $\widehat{G}(s) := \int_0^\infty e^{-sx}\,dG(x)$ the Laplace-Stieltjes transform of the claim-size distribution. Instead of only considering a single claim size distribution we consider the exponential family

$$dG_\theta(x) := \Big(\widehat{G}(-(\theta + \gamma_0))\Big)^{-1} e^{(\theta+\gamma_0)x}\,dG(x)$$

where $\theta \in \{r \in \mathbb{R} : \widehat{G}(-(r + \gamma_0)) < \infty\}$. Furthermore let $\lambda_\theta = \lambda\widehat{G}(-(\theta + \gamma_0))$. We denote by P_θ and E_θ the probability measure and expectation with respect to the law with parameter θ. Define furthermore $h_0(\theta) := h(\theta+\gamma_0) - h(\gamma_0)$ and note that the safety loading $\varrho_\theta \gtreqless 0$ iff $\theta \lesseqgtr 0$. For changing from one parameter to another the following change of measure formula is well-known (see [1] or [18])

$$P_{\theta'}[\tau \le T] = E_{\theta''}\left[\mathbb{1}_{\{\tau\le T\}} e^{(\theta'-\theta'')C_\tau - \tau(h_0(\theta')-h_0(\theta''))}\right] \tag{2}$$

where $\mathbb{1}$ denotes the indicator function. Formula (2) becomes less complicated if we choose θ' and θ'' such that $h_0(\theta') = h_0(\theta'')$. Let $(\theta^{(n)} : n \in \mathbb{N})$ be a sequence with $-\gamma_0 \le \theta^{(n)} < 0$ and denote by $\widetilde{\theta}^{(n)}$ the strictly positive solution of $h_0(\widetilde{\theta}^{(n)}) = h_0(\theta^{(n)})$. Inspired by the uncorrected diffusion approximation we set $\lambda^{(n)} := n\lambda_{\theta^{(n)}}$ and $G^{(n)}(x) := G_{\theta^{(n)}}(\sqrt{n}x)$. Now (2) can be rewritten as (see also [1])

$$\begin{aligned} P_{\theta^{(n)}}\left[\tau^{(n)} \le T\right] &= E_{\widetilde{\theta}^{(n)}}\left[\mathbb{1}_{\{\tau^{(n)}\le T\}} \exp\{(\theta^{(n)} - \widetilde{\theta}^{(n)})C^{(n)}_{\tau^{(n)}}\}\right] \\ &= E_{\widetilde{\theta}^{(n)}}\left[\mathbb{1}_{\{\tau^{(n)}\le T\}} \exp\{(\theta^{(n)} - \widetilde{\theta}^{(n)})(u + c^{(n)}\tau^{(n)} - S^{(n)}_{\tau^{(n)}})\}\right]. \end{aligned}$$

By considering expectations

$$E[S^{(n)}_{\tau^{(n)}}] = \lambda^{(n)}\mu^{(n)}E[\tau^{(n)}] = \sqrt{n}\lambda_{\theta^{(n)}}\mu_{\theta^{(n)}}E[\tau^{(n)}]$$

increases with order $\sqrt{n}$. Hence $c^{(n)}$ should increase with the same rate and it is natural to choose $\theta^{(n)} = n^{-\frac{1}{2}}\theta^{(1)}$. For a more comprehensive discussion of the approximation including asymptotic expansions see [18].
The following Proposition shows that our intuitive approach leads to a weak convergence result. The Proposition is an easy alteration of the approximation in [1, p. 44].

Proposition 2. *Let $(C_t^{(n)})$ be a sequence of classical risk processes such that $c^{(n)} = \sqrt{n}c$, $\lambda^{(n)} = \lambda n\widehat{G}(-(1-n^{-\frac{1}{2}})\gamma_0)$ and*

$$\widehat{G}^{(n)}(s) = \frac{\widehat{G}(sn^{-\frac{1}{2}} - (1-n^{-\frac{1}{2}})\gamma_0)}{\widehat{G}(-(1-n^{-\frac{1}{2}})\gamma_0)}.$$

Let (Q_t) be a (d,η^2)-Brownian motion with $d = \lambda\gamma_0\widehat{G}''(-\gamma_0)$ and $\eta^2 = \lambda\widehat{G}''(-\gamma_0)$. Then $C^{(n)} \Longrightarrow Q$. $\square$

Note that $(C_t^{(1)})$ has the same law as (C_t).
The idea of the diffusion approximation is to use the probability that the diffusion (Z_t) converges to $-\infty$ as an approximation value for the infinite time ruin probability of the risk process (X_t). As mentioned before $\lim_{n\to\infty} P[\tau^{(n)} < \infty] = P[Z_t \to -\infty]$. The stationary distribution of the overshoot for the considered model is the same as in the classical case. Therefore it makes sense to use the sequence $(C_t^{(n)})$ of Proposition 2.

We shall now compare exact values with their approximations. For simplicity we use exponentially distributed claims. The exact infinite time ruin probability in this case is given by (compare [16] or [17]):

Proposition 3. *For exponentially distributed claims with mean μ, the ruin probability for (X_t) is given by*

$$P[X_t \to -\infty] = 1 - \frac{g(X_0)}{g(\infty)}$$

where

$$g(x) = g_1(x)\mathbb{1}_{[\Delta,\infty)}(x) + g_2(x)\mathbb{1}_{[0,\Delta)}(x) + g_3(x)\mathbb{1}_{(-c/\beta_2,0)}(x)$$

and

$$g_1(x) = g_2(\Delta) + \left(\frac{\beta_1}{c}\right)^{(\lambda/\beta_1)-1} e^{c/(\beta_1\mu)} g_2'(\Delta) \int\limits_{c/\beta_1}^{x+c/\beta_1-\Delta} s^{(\lambda/\beta_1)-1} e^{-s/\mu}\, ds\,,$$

$$g_2(x) = g_3(0) + \frac{g_3'(0)}{1/\mu - \lambda/c}(1 - e^{-(1/\mu-\lambda/c)x}),$$

and

$$g_3(x) = K \int\limits_0^{x+c/\beta_2} s^{(\lambda/\beta_2)-1} e^{-s/\mu}\, ds\,,$$

with K an arbitrary constant and $\mathbb{1}$ denotes the indicator function. □

Table 1: Comparison of a risk process with exponentially distributed claims with mean $\mu = 1, \lambda = 8$ and $c = 8.2$ (i.e. $\varrho = 0.025$) with its diffusion approximation.

u	exact	approx.	Error
– 20	0.992	0.988	– 0.40 %
– 10	0.914	0.912	– 0.21 %
0	0.736	0.740	0.61 %
10	0.576	0.578	0.46 %
20	0.450	0.452	0.31 %
30	0.352	0.353	0.17 %
40	0.275	0.275	0.03 %
50	0.215	0.214	– 0.11 %
60	0.167	0.167	– 0.25 %
70	0.130	0.130	– 0.38 %
80	0.101	0.101	– 0.51 %
90	0.0784	0.0779	– 0.63 %
100	0.0605	0.0601	– 0.75 %
110	0.0466	0.0462	– 0.86 %
120	0.0356	0.0353	– 0.97 %
130	0.0270	0.0268	– 1.06 %
140	0.0203	0.0201	– 1.13 %
150	0.0151	0.0149	– 1.19 %
160	0.0109	0.0108	– 1.21 %
170	0.00770	0.00761	– 1.18 %

Table 2: Comparison of a risk process with exponentially distributed claims with mean $\mu = 0.909, \lambda = 10$ and $c = 9.5$ (i.e. $\varrho = 0.045$) with its diffusion approximation.

u	exact	approx.	Error
– 20	0.973	0.963	– 0.97 %
– 10	0.811	0.806	– 0.59 %
0	0.531	0.534	0.52 %
10	0.331	0.331	0.00 %
20	0.206	0.205	– 0.52 %
30	0.128	0.127	– 1.03 %
40	0.0799	0.0786	– 1.55 %
50	0.0497	0.0487	– 2.06 %
60	0.0310	0.0302	– 2.57 %
70	0.0193	0.0187	– 3.07 %
80	0.0120	0.0116	– 3.57 %
90	0.00746	0.00715	– 4.06 %
100	0.00464	0.00442	– 4.55 %
110	0.00288	0.00273	– 5.03 %
120	0.00178	0.00169	– 5.50 %
130	0.00110	0.00104	– 5.96 %
140	0.000679	0.000635	– 6.40 %
150	0.000415	0.000386	– 6.81 %
160	0.000250	0.000232	– 7.18 %
170	0.000148	0.000137	– 7.49 %

For exponentially distributed claims the parameters of the model have the following values: $\widehat{G}(s) = (1 + s\mu)^{-1}, \gamma_0 = \mu^{-1}(1 - \sqrt{\lambda\mu/c})$ (the latter because $\widehat{G}(-s)$ is not defined for $s \geq \mu^{-1}$). This leads to

$$\widehat{G}''(-\gamma_0) = 2\frac{c}{\lambda}\sqrt{\frac{c\mu}{\lambda}}$$

and therefore the parameters of the diffusion are

$$d = 2c\left(\sqrt{\frac{c}{\lambda\mu}} - 1\right) \quad \text{and} \quad \eta^2 = 2c\mu\sqrt{\frac{c}{\lambda\mu}} .$$

Table 3: Comparison of a risk process with exponentially distributed claims with mean $\mu = 1.25$, $\lambda = 3$ and $c = 4$ (i.e. $\varrho = 0.067$) with its diffusion approximation.

u	exact	approx.	Error
−20	0.999	0.994	−0.48 %
−10	0.913	0.900	−1.38 %
0	0.599	0.606	1.14 %
10	0.363	0.364	0.33 %
20	0.220	0.219	−0.48 %
30	0.134	0.132	−1.28 %
40	0.0810	0.0794	−2.06 %
50	0.0492	0.0478	−2.85 %
60	0.0298	0.0287	−3.63 %
70	0.0181	0.0173	−4.40 %
80	0.0110	0.0104	−5.16 %
90	0.00664	0.00625	−5.92 %
100	0.00402	0.00375	−6.66 %
110	0.00243	0.00225	−7.39 %
120	0.00147	0.00135	−8.10 %
130	0.000884	0.000807	−8.79 %
140	0.000530	0.000480	−9.45 %
150	0.000315	0.000283	−10.07 %
160	0.000185	0.000165	−10.63 %
170	0.000106	0.0000940	−11.09 %

Table 4: Comparison of a risk process with exponentially distributed claims with mean $\mu = 1.25$, $\lambda = 1.1$ and $c = 1.5$ (i.e. $\varrho = 0.091$) with its diffusion approximation.

u	exact	approx.	Error
− 20	1.000	1.000	− 0.00 %
− 10	0.998	0.984	− 1.33 %
0	0.652	0.664	1.96 %
10	0.334	0.336	0.49 %
20	0.172	0.170	− 0.95 %
30	0.0882	0.0861	− 2.38 %
40	0.0453	0.0436	− 3.78 %
50	0.0232	0.0220	− 5.17 %
60	0.0119	0.0112	− 6.53 %
70	0.00613	0.00564	− 7.88 %
80	0.00314	0.00286	− 9.20 %
90	0.00161	0.00144	− 10.51 %
100	0.000828	0.000731	− 11.79 %
110	0.000425	0.000370	− 13.05 %
120	0.000218	0.000187	− 14.28 %
130	0.000112	$9.43\ 10^{-5}$	− 15.49 %
140	$5.69\ 10^{-5}$	$4.74\ 10^{-5}$	− 16.65 %
150	$2.89\ 10^{-5}$	$2.38\ 10^{-5}$	− 17.76 %
160	$1.45\ 10^{-5}$	$1.18\ 10^{-5}$	− 18.79 %
170	$7.11\ 10^{-6}$	$5.71\ 10^{-6}$	− 19.70 %

Note that $\eta^2 > 2\lambda\mu^2$ in the case of a positive safety loading and therefore the variance of the Brownian motion in a unit time interval is larger than for the approximation used in [16].

The Tables 1−4 show examples with forces of interest $\beta_1 = 0.058$ (interest rate 6.0 %) for invested money, $\beta_2 = 0.095$ (10.0 %) for borrowed money and liquid reserve barrier $\Delta = 200$. Remember, that u denotes the initial capital. The relative error is the quantity

$$\frac{\text{approximation} - \text{exact value}}{\text{exact value}} 100\,\%\,.$$

Table 1 and Table 2 are the examples also discussed in [16]. A comparison shows that the error for the corrected diffusion approximation is less than half the error of the approximation used in [16]. Note that in all four examples the relative error lies under 11 % for $P[\tau < \infty] < 10^{-3}$ and it is even smaller if the safety loading is smaller as in the example of Table 4. The claim arrival intensity λ is rather small in the cases of Table 3 and 4. Because $\lambda^{(n)} \to \infty$ as $n \to \infty$ one may expect that the approximation fits better if λ is chosen larger. But playing with the parameters it turns out that λ does not hardly influence the fit of the approximation. It was necessary to choose λ small because otherwise $P[\tau < \infty]$ decreases too quickly as u increases (see Table 4).

These examples show that the approximation gives a 'good' estimate of the real ruin probability for the most interesting values of u. If the distribution function of the claims and the claim arrival rate are known, it is easy to calculate a rude estimate of the ruin probabilities, which tells us for instance how quickly the ruin probabilities go to zero. The disadvantage of the method is that it yields no bounds for the ruin probability. So the method is only suitable in order to get an impression where the exact values lie.

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Summary

In many situations in insurance risk theory one has the problem that exact values for the ruin probability are hard to obtain. Therefore approximations are called for. It turns out that to get crude estimates for the ruin probability the so called diffusion approximations are useful. Recently a method to combine diffusion approximations for the classical Cramér-Lundberg model and diffusion approximations for more general models was worked out ([16]). In the present paper it is shown that the method can be improved by using corrected diffusion approximations.

Zusammenfassung

In der Risikotheorie steht man oft vor dem Problem, dass exakte Werte für die Ruinwahrscheinlichkeit nur sehr schwer zu erhalten sind. Man versucht sich deshalb mit Approximationsmethoden auszuhelfen. Um grobe Schätzwerte für die Ruinwahrscheinlichkeiten zu erhalten, stellen die sogenannten Diffusionsapproximationen eine nützliche Methode dar. Kürzlich wurde eine Methode entwickelt ([16]), die es erlaubt, Diffusionsapproximationen für das klassische Cramér-Lundberg Modell und Diffusionsapproximationen für allgemeinere Prozesse zu kombinieren. In diesem Artikel wird gezeigt, dass eine Verbesserung der Approximation erreicht werden kann, indem man korrigierte Diffusionsapproximationen benutzt.

Résumé

Dans la théorie du risque on est souvent confronté au problème, que les valeurs exactes de la probabilité de ruine sont très difficiles à obtenir. On cherche alors à utiliser des méthodes d'approximation. Les approximations par processus de diffusion se révellent être une méthode efficace pour obtenir des estimations grossières de la probabilité de ruine. Une méthode développée récemment [16] permet de combiner des approximations par processus de diffusion pour le modèle classique de Cràmer-Lundberg et des approximations par processus de diffusion pour des processus plus généraux. Dans ce papier on montre que l'approximation peut être améliorée en utilisant des approximations par processus de diffusion corrigées.