Modelling of risks in insurance groups for the Swiss Solvency Test

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1 Introduction

The legal form of some insurance undertakings is that of a standalone company, whereas others are organised in groups. Operating either in the same jurisdiction or in different countries, the companies of a group often act under the same brand and name.

For the purpose of this paper an insurance group is a set of legal entities which are bound by some type of ownership between the entities. At least some of them have to run insurance operations. In addition, the description of the group contains the information about the existence and properties of capital and risk transfer instruments between any two members of the group. Examples of risk transfer instruments are financial guarantees and reinsurance agreements.

In practice, the structure of ownership of many groups is tree like. However, often for historical reasons, other groups have got circular connections. For instance, a superior company might own 60% of a subsidiary A and 100% of a subsidiary B, while B is the owner of the remaining 40% of A.

The topological structure of risk transfer instruments in the group can differ from the structure of ownership. Some groups transfer mayor parts of the risks out of the subsidiaries and concentrate them in a hierarchically superior legal entity. This helps to reduce outwards premiums for reinsurance protection or simply to reduce the risk burden on the subordinated legal entities. Other groups diminish their tax obligations by transferring risks and therefore also expected profits from one jurisdiction to a place which offers lower fiscal burden. This is not necessarily the domicile of the parent company.

2 SST Group Modelling

2.1 *Value and Risks in Insurance Groups*

To measure the value of a group, effects of double counting have to be handled with care. Double counting can be disturbing the picture if some type of intra group loans or cross participations exists in the group. Usually, to overcome the problem, a group consolidated balance sheet is used. In essence, this is a catalogue or a list of all individual assets and liabilities existing somewhere in the group. The consolidated balance eliminates assets and liabilities which exist only between group members by netting them out. What remains are assets and liabilities to external counterparties. All assets are considered to be usable to cover the sum of all liabilities.

Some insurance groups define their group risk as the risk of possible changes in their consolidated balance sheet, because the consolidated balance sheet measures the value of the group. However, the result of this definition of risk is associated with a number of disadvantages.

A fundamental characteristic of such risk measuring is the assumption that adverse changes of each asset or liabilities can be offset with values of all other assets and liabilities of the consolidated balance sheet. This assumption originates from the fact that by consolidation, all financial positions are lumped together in one large basket. Information about the position of assets and liabilities within the group are forgotten.

In reality, a loss does not occur in a large group basket but in a legal entity which merely is a part of the whole group. Of course a group will in many circumstances transfer capital into the specific legal entity in order to help to carry a huge loss. However, firstly, there might be conditions that limit the fungibility of capital, i.e. there might be restrictions on transferring assets out of another part of the group into the suffering entity. Secondly, as long as the group management has no legal obligation to support a subsidiary with capital in case of distress, it might just not want to transfer capital for economic reasons. In both cases, the loss in the subsidiary is not covered by the overall mass of group assets. This is not reflected when risk is measured on the basis of a consolidated balance sheet.

In addition insurance groups are highly interested in how to allocate capital to legal entities. Providing the answer requires applying the group structure and an allocation method on a group risk result which was calculated by neglecting that group structure. It turns out that the final allocation of capital depends heavily on the actual choice of the allocation method.

For these reasons the Swiss Solvency Test (SST) follows another path to define group risk modelling. This way of modelling is based upon three points. Firstly liabilities to third parties are obligations of legal entities but not of the group and that each legal entity pays only for those obligations which it really has to pay for due to binding contracts. Secondly a group is a collections of legal entities together with interrelations consisting of ownership and of formal risk transfer agreements; and, thirdly, owning a subsidiary means holding an asset which is assigned a current market value and a future but uncertain value.

The first point states that being part of a group does not include for a company being able to count on financial help in case of distress just because this company is part of a group. Another member of the group would have to provide this help. The first point states that no group member would pay if it is not forced by a contract. In particular, this implies that a parent company is in the position of a shareholder with limited liability towards its subsidiaries. If one of them is bankrupt, the parent is not forced to inject capital per se.

At first sight this puts each group member in a standalone position. However, the second point implies that existing risk transfer instruments such as financial guarantees or reinsurance agreements shall be fully taken into account. It is those instruments that provide the exact conditions in which cases how much capital has to be moved from one group member to another. The SST does not consider the group as a monolithic block but as a collection of entities which behave in a way which is optimal given the laws of the game, in particular the risk transfer instruments.

It is important that the risk transfer instruments are legally binding. Firstly this includes that they must have been accepted and agreed on by regulators involved which otherwise might stop a group internal transfer of assets out of the regulators sphere of influence. Secondly, being legally binding means that an oral promise by the group top management to support subsidiaries is not sufficient to be taken into account as a risk transfer instrument.

The third point is related to the question how the financial position of a parent company has to be modelled. The SST considers owing a subsidiary as holding an asset of a given value at evaluation date and uncertainty in future value, which then is part of the risk modelling. In addition the third point is also related to the question of limited capital fungibility within the group.

Following the fundamental principles of the SST, all assets have to be modelled market consistently. Therefore the value of a subsidiary for a parent company is the market value, i.e. the transfer price which two willing and knowledgeable parties would agree on. Clearly a willing buyer does not only pay for the value of all financial positions in a subsidiary but also for discounted expected profits related to future business. Since the risk measurement of the parent includes modelling future values of all assets and liabilities, also the possible future values of subsidiaries have to be treated stochastically. If subsidiaries are valued with market value including future profits, these future profits have also to be part of the risk modelling.

However, what is usually done is valuing subsidiaries with their run off value, which is the difference of market consistent prices for the subsidiary's assets minus market consistent values of liabilities. By construction, this difference does not include profits of future business. Modelling the uncertainty of that quantity as the value of a subsidiary is substantially easier than the supplemental modelling of uncertainty in future business.

If a risk taking transfer instrument exceeds the amount of fungible assets of an entity, correctly taking into account the limited fungibility implies that the exceeding part is modelled as a part of the credit risk of the receiver of the other entity. Nevertheless, a subsidiary of a parent company is considered as fungible assets for the parent. Indeed, it might be hardly possible to transfer capital out of a subsidiary. However, it is always possible to sell a subsidiary for its market value, i.e. to transform it into cash. This means that a parent company can provide a support for its subsidiary A by selling subsidiary B, even if the capital within B is not fungible.

Modelling the risk of a group under the SST means simultaneously modelling the financial position, i.e. assets and liabilities, of each legal entity over a one year time horizon taking into account all risk transfer agreements. As a result the financial position of each legal entity should be known for each state of the world. The final result will be the joint probability distribution of assets minus liabilities of each legal entity.

2.2 *Group Diversification*

An important property of insurance groups is risk diversification. In general, risk diversification comes into play if two or more risky positions are considered in combination. For instance this might by a bet on a stock combined with an earthquake insurance contract. Given that these positions are not completely dependent it is less probable that both positions show adverse developments than that the outcome of only one of the position is bad.

The concept of risk diversification requires the concept of quantification of a risk, i.e. a quantitative risk measure. Many of these have been discussed; VaR, TailVaR or standard deviation being only the most prominent ones. For some purposes even the expected value can be considered as a risk measure.

The risk diversification related to a set of risky positions is the difference between (i) the arithmetic sum of risks of the individual positions and (ii) the risk of the combination of the positions. If it is the expected value which is used as risk measure, putting together risky positions does not reduce the risk. For the other examples of risk measures, however, combining positions reduces the risk in many cases.

The existence of risk diversification is separated from the question who the beneficiary of the diversification is. An economic benefit from risk diversification of two or more risky positions only arises if someone combines these positions in its portfolio. As a first example we consider two risky positions, the first one being a French wine cellar exposed to fire and the second one a construction risk in Brazil. Obviously although these two risks diversify there is probably nobody who is exposed to them at the same time. Hence nobody benefits from the diversification effect. A second example is a shareholder owning a share from company A and another one from company B. The shareholder measures risk using the TailVaR of the distribution of changes in values over a given time period. For the sake of argument we assume that the future values of these positions are not totally dependent, hence there is a risk diversification. The shareholder, but neither A or B, is the beneficiary of that diversification effect.

The SST Group modelling takes group diversification effects completely into account and allocates them correctly to the legal entities benefiting from group diversification. Risk diversification between risks in subsidiaries (e.g. earthquake in Japan and disability in Canada) is expressed in diversification between the values of the subsidiaries. Hence, the beneficiary of such diversification is the owner of subsidiaries. Benefiting from diversification means to combine risks in a portfolio which is then less risky than the sum of standalone risks. Clearly the subsidiaries are not in such a position. However, they can benefit from being part of a group by getting a reinsurance cover or a financial guarantee from other group members.

2.3 *Examples*

We present three examples in order to show the effects of group diversification and of a group internal financial guarantee. In all examples the group is composed of a parent company Pa holding two subsidiaries S1 and S2 as its only assets. The universe of financial positions consists of three assets A1, A2 and A3; and two liabilities L1 and L2. For simplicity, all five variables are supposed to be iid, with A1 $\sim$ N(1; 0.2). Which asset and liability are hold by which entity is shown in table T1. Note that the parent has no liabilities.

The first example serves as a base case to which the second and third example can be compared. In that base case, S1 and S2 hold different assets and different liabilities, which makes their values stochastically independent. No financial guarantee does exist in the base case.

The second example is characterised by a lower diversification between subsidiaries S1 and S2. This is achieved by introducing an overlap in the asset allocations of S1 and S2.

In the third example, the dependence structure is as in example 1; however, the parent company has given a financial guarantee for subsidiary 1. In this guarantee, the parent promises to inject capital into S1 if its assets should not exceed the liabilities anymore.

T1: Number of Assets and liabilities hold by legal entities in examples 1, 2, and 3. The only difference in the allocations between the examples is subsidiary S2 in example 2 holding asset A1 instead of A2. Since A1 is also hold by S1, the subsidiaries become stochastically dependent in example 2.

Example 1	**A1**	**A2**	**A3**	**L1**	**L2**
Parent Company					
Subsidiary 1	1.6			1	
Subsidiary 2		1	0.6		1

Example 2	**A1**	**A2**	**A3**	**L1**	**L2**
Parent Company					
Subsidiary 1	1.6			1	
Subsidiary 2	1		0.6		1

Example 3	**A1**	**A2**	**A3**	**L1**	**L2**
Parent Company					
Subsidiary 1	1.6			1	
Subsidiary 2		1	0.6		1

The examples are evaluated using Monte Carlo simulation. The variables A1 to L2 are simulated 2 million times to obtain 2 million possible values of the three legal entities Pa, S1, and S2.
The results are given in table T2 in terms of expectation value, VaR and TailVar, both at the 1% level. The results could have also been obtained analytically as long as normal distributions and only simple dependency structures are concerned. However, Monte Carlo was preferred due to its expandability to more complex setups.
Observations regarding the base case are: (i) expected values of S1 and S2 are identical. In view of their assets and liabilities shown in T1, this is reasonable. (ii) Risk, if measured in VaR or TailVar, is larger for S1 than for S2. This is explained by the fact that S2 benefits from a diversification between asset A2 and A3, whereas S1 has no diversification benefit on the asset side. (iii) Expected value of the parent company is larger than the sum of expected values of S1 and

S2. This is an effect of the limited liability of the parent towards its subsidiaries, i.e. the fact the parent company cannot lose more than the initial value of an asset, in particular of subsidiaries S1 and S2. For instance, the value of S1 from the point of view of the parent is modelled as max(S1, 0). If the value of S1 is negative, subsidiary 1 is bankrupt.
Density distributions for Pa, S1, and S2 are shown in Figure 1. In Figure 2, 2000 samples out of the 2 million Monte Carlo simulations are drawn showing the values of S1 and S2 together with the corresponding values for the parent company.
Comparing example 2 with the base case, we observe that nothing has changed for subsidiaries S1 and S2, even if they are correlated now. The reduction in diversification impacts purely on the parent company, of which the risk increases. Graphical results for example 2 are shown if Figures 3 and 4.
In the third example the risk transfer effect of the parental guarantee for S1 becomes visible. The risk of S1 is lower than in the base case. Conversely, the parent's risk increases. This can be seen in table T2 and in Figure 5. Interestingly, in terms of VaR, the increase of risk for the parent is smaller than due to the reduction in diversification in example 2. However if risk is measured with TailVaR, the risk increase due to the guarantee is larger then in example 2.
The guarantee is associated with a credit risk for subsidiary S1. The amount which can be transferred to from Pa to S1 cannot exceed the amount of the other asset of Pa, namely S2. Therefore, if the guarantee is invoked, the parent company might be unable to pay.

T2: Expected value, VaR(1%), and TailVaR(1%) of the values of the subsidiaries S1, S2, and the parent company.

	Example 1 base case	Example 2 Reduced diversification	Example 3 guarantee from Pa to S1
E[S1]	0.60	0.60	0.61
VaR(S1)	−0.28	−0.28	0.00
TailVar(S1)	−0.41	−0.41	−0.08
E[S2]	0.60	0.60	0.60
VaR(S2)	−0.12	−0.12	−0.12
TailVar(S2)	−0.22	−0.22	−0.22
E[Pa]	1.21	1.21	1.20
VaR(Pa)	0.20	0.01	0.10
TailVar(Pa)	0.11	0.00	−0.05

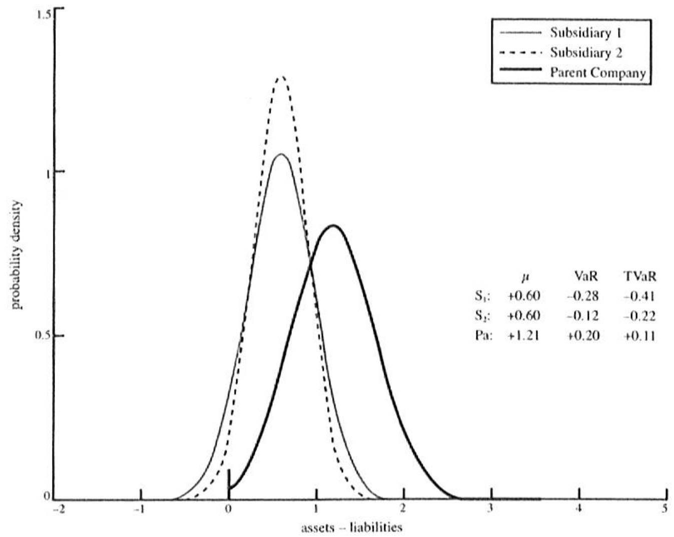

Figure 1 Histogram of assets – liabilities of subsidiary 1 and 2, and parent company in example 1 produced by the Monte Carlo Method. Bin size is 0.02. Note that the value of the parent never is negative due to limited liability towards the subsidiaries. Therefore, the density of the parent has got an atom or Dirac-Delta contribution at A-L = 0. While expected values of S1 and S2 are equal, S1 is riskier than S2.

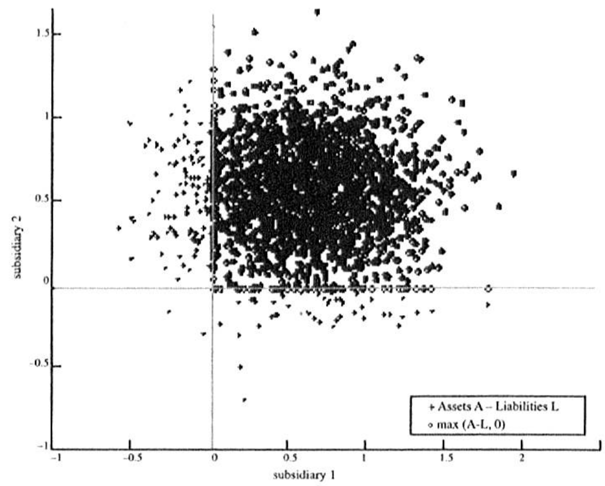

Figure 2 2000 base case sample points of the Monte Carlo simulation. Plus signs are the values of assets - liabilities of subsidiaries 1 and 2. Open circles denote the value of S1 and S2 from the point of view of the holding company. Due to limited liability, these values cannot be negative.

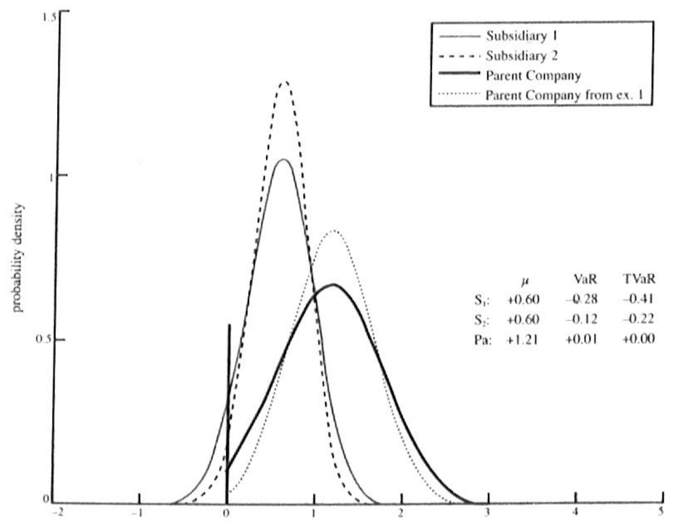

Figure 3 Probability density of S1, S2, and Pa in the correlated case of example 2. The distributions of A-L of S1 and S2 are the same as in the base case (Figure 1). The distribution of the parent (solid thick line) has substantially widened compared to the base case (dotted thick line).

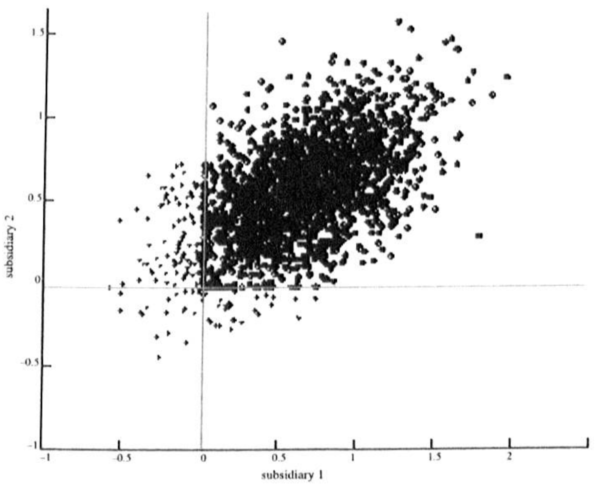

Figure 4 A-L of subsidiaries (plus signs) and values of subsidiaries for parent (open circles) as in Figure 2.

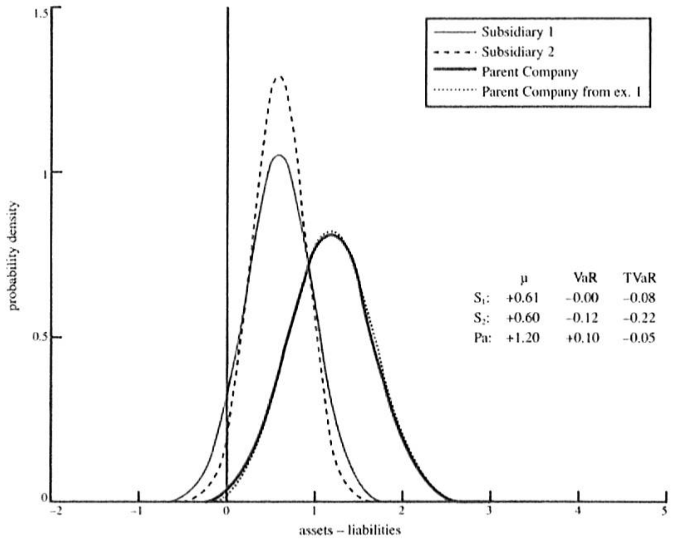

Figure 5 Probability density of subsidiaries S1 and S2, and of parent company in the case of the financial guarantee for S1. In many cases of negative outcomes, S1 is lifted to zero level, which corresponds to the peak at A-L = 0. Therefore, the lower tail is drastically shorter than in the base case (see e.g. Figure 1). However, negative values for S1 still do exits, since the parent is not able to pay in all situations.

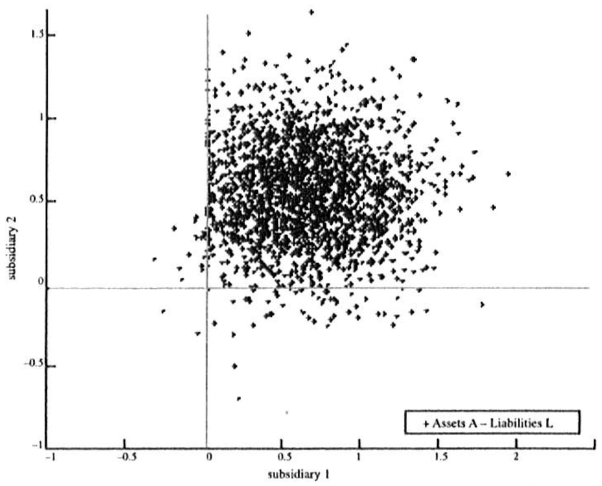

Figure 6 A-L for subsidiaries S1 and S2 in the case of the financial guarantee from the parent to S1. The concentration on the y-axis is the effect of the parental support in adverse situations of S1. However, the lower the value of S2, the less is the parent able to fulfil the guarantee in case of distress. Therefore negative values for S1 are rare but still possible.

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Abstract

We have presented how to model an insurance group in the SST. This way of modelling does not treat the group as one monolithic set of assets and liabilities, but produces results for each group member entity and takes into account the relations such as ownership and risk transfer instruments between these entities. The theoretical considerations have been applied in a quantitative toy model of a simple group structure.

Zusammenfassung

Wir haben gezeigt, wie eine Versicherungsgruppe im SST modelliert werden kann. Diese Art der Modellierung behandelt die Gruppe nicht als konsolidierte Menge aus Assets und Verpflichtungen, sondern sie erzeugt Ergebnisse für jede Gruppeneinheit und berücksichtigt Beziehungen wie Besitzverhältnisse und Risikotransfer-Instrumente zwischen diesen Einheiten. Die theoretischen Überlegungen sind auf ein Beispielmodell mit einer einfachen Gruppestruktur angewandt worden.

Résumé

Nous avons montré comment un groupe d'assurance peut être modélisé dans le SST. Notre modèle ne considère pas le groupe comme un ensemble monolithique de biens et d'engagements, il produit des résultats pour chaque entité du groupe et prend en compte les relations telles que les rapports de possession et les instruments de transfert de risques entre ces entités. Les considérations théoriques sont appliquées à un exemple avec une structure de groupe simple.