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Case Study on the optimality of reinsurance contracts

1. Introduction

A main goal of reinsurance consists to protect an insurance company, hereafter called the cedent, against large losses caused either by excessively large claims or by a large number of claims. A reinsurance contract determines the rules according to which premium payments and unearned premium reserves, as well as claim payments, case reserves and IBNR reserves are split between the ceding and the reinsurance companies. A fundamental problem is to determine conditions for which a reinsurance contract (that operates within a specified insurance risk model) provides a less risky position to the reinsurer and/or to the cedent. It is therefore important to develop methods that compare the riskiness of different reinsurance contracts and that select optimal ones. An outline of the case study follows.
Section 2 recalls the definitions of the basic reinsurance contracts, namely quota share, surplus, excess-of-loss and stop-loss reinsurance and some variants of them. We assume that the aggregate claims random variable is compound Poisson distributed.
In recent years, the analysis of Glineur and Walhin(2006) as well as Lampaert and Walhin(2005) about the optimality of reinsurance within the class of proportional reinsurance contracts has led to quite interesting results. Our purpose is the extension of this analysis to a broader class of reinsurance contracts including some basic non-proportional reinsurance contracts. The optimization of reinsurance is based on the following two accepted criteria:

- the *de Finetti criterion*, introduced in de Finetti(1940), which consists of minimizing the variance of the retained risk for fixed expected retained profit
- the *RORAC criterion*, which maximizes the return on risk adjusted capital of the retained risk

Without providing a general proof, the detailed analytical and numerical comparisons made in Section 3 and 4 lead us to the following main observations:

a) For *fixed expected retained aggregate claims*, non-proportional reinsurance is more efficient than proportional reinsurance as measured in terms of the reduction in the coefficient of variation of the retained aggregate claims. Moreover, for higher fixed expected retained aggregate claims (variable) excess-of-loss may be more efficient than stop-loss, which contradicts the traditional belief that stop-loss reinsurance is an optimal form (e. g. Borch(1960), Kahn(1961), Arrow(1963/74), Ohlin(1969), Pesonen(1983), Hesselager(1993), Wang(1995)). However, note that the known proofs of stop-loss optimality assume alternative reinsurance forms, which are defined in terms of aggregate losses and not in terms of individual losses as for excess-of-loss.

b) For *constant expected retained profit* and *fixed reinsurance loading factor*, the same comparative results hold when maximizing the RORAC measure within an approximate Gamma distributed model of the retained aggregate claims.

The Appendix offers inequalities about the loading factors of the used expected value premium principle, which turns out to be very useful in the discussion of the numerical examples.

2. Basic Reinsurance Contract Types

The basic reinsurance forms are quota share, surplus, excess-of-loss and stop-loss contracts and variants of them like variable quota-share, table of lines surplus, variable excess-of-loss and variable stop-loss. Quota-share and stop-loss provide some protection against frequency risk if the number of claims is large while surplus and excess-of-loss may protect against severity risk if the claim sizes are large. Extreme value reinsurance, which is not considered, is discussed in papers from Ammeter(1964) and Thépaut(1950) to Silvestrov et al.(2006).
We assume that the aggregate claims of an insurance risk portfolio over some fixed time period can be represented by a compound Poisson random variable

$$S = \sum_{i=1}^{N} X_i, \tag{2.1}$$

where the *number of claims* N is Poisson (λ) distributed, the non-negative *claim sizes* $X_i's$ are independent, identically distributed and independent from N. The identical random variables are denoted $X =_d X_i, \quad i = 1, \dots, N$. The *reinsured aggregates claims* are denoted by S_r while the retained *aggregate claims of the ceding company* are denoted by S_c and one has the risk decomposition $S = S_c + S_r$. A description follows for each basic reinsurance contract.

Quota Share

The cession rate is the same across the whole insurance portfolio. For each individual claim, the reinsurer covers a fixed proportion q, called *quota rate*. One has the expressions

$$S_c = (1-q) \cdot S, \quad S_r = q \cdot S. \tag{2.2}$$

Surplus

The cession rate is a function of both the sum insured SI_i associated to the policy hit by the i-th individual claim and a quantity called the line of retention. The *line* R is the maximal amount that the insurer is willing to pay in case of a loss (for each policy in the portfolio). To ensure that the maximal loss will never exceed the line, the retention rate must be equal to

$$\min\left(1, \frac{R}{SI_i}\right) \in [0,1]. \tag{2.3}$$

The reason for the min operator is that when the sum insured is smaller than the line, i.e. when R exceeds SI_i, the retention rate has to be equal to 1. The cession rate must then be defined as

$$1 - \min\left(1, \frac{R}{SI_i}\right) = \left(1 - \frac{R}{SI_i}\right)_+, \tag{2.4}$$

where $(x)_+ = \max(0, x)$. The risk decomposition of a surplus reinsurance depends on the individual claims as follows:

$$S_c = \sum_{i=1}^{N} \min\left(1, \frac{R}{SI_i}\right) \cdot X_i, \quad S_r = \sum_{i=1}^{N} \left(1 - \frac{R}{SI_i}\right)_+ \cdot X_i, \tag{2.5}$$

Excess-of-loss reinsurance

An excess-of-loss or XL contract is determined by the *deductible* $d > 0$. The reinsurer pays that part of each claim that exceeds the deductible. The risk decomposition between the ceding company and the reinsurer is determined by the formulas:

$$S_c = \sum_{i=1}^{N} \min(X_i, d), \quad S_r = \sum_{i=1}^{N} (X_i - d)_+. \tag{2.6}$$

Stop-loss reinsurance

A stop-loss or SL contract is determined by the *priority* $L > 0$. The reinsurer pays any part of the incurred aggregate claims that exceeds the priority. The risk decomposition reads

$$S_c = \min(S, L), \quad S_r = (S - L)_+. \tag{2.7}$$

Variable Quota Share

In a *variable quota share* the portfolio is partitioned into several segments for which an individual cession rate applies. Suppose the portfolio is divided into m segments and in each segment a collective model of risk theory applies such that the aggregate claims in the segments and portfolio are given by

$$S_j = \sum_{i=1}^{N_j} X_{ji}, \quad j = 1, 2, \ldots, m, \quad S = \sum_{j=1}^{m} S_j. \tag{2.8}$$

The risk decomposition between cedent and reinsurer is then given by

$$S_c = \sum_{j=1}^{m} (1 - q_j) \cdot S_j, \quad S_r = \sum_{j=1}^{m} q_j \cdot S_j. \tag{2.9}$$

Table of Lines Surplus

In a *table of lines surplus* the portfolio is partitioned into segments for which the same line applies. Under the segmentation (2.8), assume that the sum insured SL_{ji} is associated to the policy hit by the i-th individual claim X_{ji} in the j-th segment. Then one has the decomposition

$$S_c = \sum_{j=1}^{m} \sum_{i=1}^{N_j} \min\left(1, \frac{R_j}{SI_{ji}}\right) \cdot X_{ji}, \quad S_r = \sum_{j=1}^{m} \sum_{i=1}^{N} \left(1 - \frac{R_j}{SI_{ji}}\right)_+ \cdot X_{ji}. \tag{2.10}$$

Variable Excess-of-loss

Under the segmentation (2.8) and a deductible d_j for the j-th segment, one has

$$S_c = \sum_{j=1}^{m} \sum_{i=1}^{N_j} \min(X_{ji}, d_j), \quad S_r = \sum_{j=1}^{m} \sum_{i=1}^{N} (X_{ji} - d_j)_+. \tag{2.11}$$

Variable Stop-loss

Under the segmentation (2.8) and a priority L_j for the j-th segment, one has

$$S_c = \sum_{j=1}^{m} \min(S_j, L_j), \quad S_r = \sum_{j=1}^{m} (S_j - L_j)_+. \tag{2.12}$$

Remark 2.1. There exists a huge of possible variations and combinations of the above basic contract types, whose optimal properties are not analyzed in the present case study. Among them one may mention excess-of-loss and stop-loss layer contracts, excess-of-loss layers with aggregate layers, chains of excess-of-loss layers with stop-loss limits, combination of quota share and surplus with excess-of-loss, linear combination of quota share and stop-loss, linear combination of stop-loss in two layers, truncated excess-of-loss and stop-loss, and so on.

3. Optimal Reinsurance

It is well-known from the literature that non-proportional reinsurance like excess-of-loss and stop-loss covers are more efficient than proportional covers like quota share and surplus. Despite this fact, there exist numerous reasons for the persistence of proportional reinsurance in practice like moral hazard behavior after stop-loss reinsurance, pricing difficulties and high reinsurance loadings in non-proportional reinsurance. Recently, Glineur and Walhin(2006) as well as Lampaert and Walhin(2005) have considered the optimality of reinsurance within the class of proportional reinsurance contracts. We extend this analysis to a broader class including non-proportional reinsurance. Optimization is performed with respect to the de Finetti and RORAC criteria and the expected value principle for premium calculation. The mathematical formulation requires the following quantities and definitions:

$S,\ S_r,\ S_c:$ aggregate claims, reinsured claims, retained claims of cedent

$\mu = E[S],\ \mu_r = E[S_r],\ \mu_c = E[S_c]$: expected values of $S,\ S_r,\ S_c$

$\theta_r^*,\ \theta_c:$ loading factors of the reinsurer and cedent

$P = (1+\theta_c)\mu,\ P_r = (1+\theta_r)\mu_r,\ P_c = P - P_r$: premium, reinsured premium, retained premium

$G_c = P_c - S_c:$ retained profit of the cedent

$VaR_\alpha[X]:$ value-at-risk of the random variable X to the confidence level α

$CVaR_\alpha[X] = E[X|X > VaR_\alpha[X]]$: conditional value-at-risk of the random variable X (with continuous distribution function) to the confidence level α

$RAC_\alpha[G_c] = CVaR_\alpha[-G_c] = CVaR_\alpha[S_c] - P_c$: risk adjusted capital of the cedent to the confidence level α

$RORAC_\alpha[G_c] = \frac{E[G_c]}{RAC_\alpha[G_c]} = \frac{P_c - \mu_c}{RAC_\alpha[G_c]}$: return on risk adjusted capital of the cedent

The considered optimization criteria translate as follows.

de Finetti criterion

$\sigma_C^2 = Var[S_c] = \min$. under the constraint $E[G_c] = (\theta_c - \theta_r)\mu + \theta_r \mu_c = const$

RORAC criterion

$RORAC_\alpha[G_c] = \max$. under the constraints $E[G_c] = const$ and $\theta_r = const$

For each basic reinsurance contract type considered in Section 2, we list below the required formulas and model assumptions under which these optimization criteria are applied.

Quota Share

$$q = 1 - \frac{E[G_c] - (\theta_c - \theta_r)\mu}{\theta_r \mu} = \frac{\theta_c \mu - E[G_c]}{\theta_r \mu} \in [0, 1] \tag{3.1}$$

Surplus

We *assume* that the ratios $\tau_i = \frac{R}{SI_i}$, $i = 1, \ldots, N$, are independent and identically log-normally distributed random variables $\tau_i =_d \tau$ that have location and scale parameters μ_τ, σ_τ such that

$$\pi_\tau = E\big[(1-\tau)_+\big] = \bar{\Phi}\left(\frac{\mu_\tau}{\sigma_\tau}\right) - e^{\mu_\tau + \frac{1}{2}\sigma_\tau^2} \cdot \bar{\Phi}\left(\frac{\mu_\tau}{\sigma_\tau} + \sigma_\tau\right) \tag{3.2}$$

$$\pi_{2,\tau} = E\Big[(1-\tau)_+^2\Big] = \bar{\Phi}\left(\frac{\mu_\tau}{\sigma_\tau}\right) - 2e^{\mu_\tau + \frac{1}{2}\sigma_\tau^2} \cdot \bar{\Phi}\left(\frac{\mu_\tau}{\sigma_\tau} + \sigma_\tau\right)$$

$$+ \, e^{2(\mu_\tau + \sigma_\tau^2)} \cdot \bar{\Phi}\left(\frac{\mu_\tau}{\sigma_\tau} + 2\sigma_\tau\right) \tag{3.3}$$

with $\bar{\Phi}(x) = 1 - \Phi(x)$, $\Phi(x)$ the standard normal distribution. In practice, this assumption is justified by the fact that the line of surplus R and the variable insured sums SI_i are known and consequently it is possible to estimate μ_τ, σ_τ from the equations

$$E\left[\frac{R}{SI_i}\right] = e^{\mu_T + \frac{1}{2}\sigma_T^2}, \quad Var\left[\frac{R}{SI_i}\right] = \left(e^{\sigma_T^2} - 1\right) \cdot E\left[\frac{R}{SI_i}\right]^2. \tag{3.4}$$

The (optimal) mean surplus cession coincides with the (optimal) quota cession

$$\pi_T = \frac{\theta_c \mu - E[G_c]}{\theta_r \mu} \in [0,1]. \tag{3.5}$$

Variable Quota Share

According to de Finetti(1940) (see also Lampaert and Walhin(2005)) the optimal quota share rates are given by (notations of Section 2)

$$1 - q_j = \min\left\{\xi\theta_r \frac{\mu_j}{\sigma_j^2}, 1\right\}, \quad \mu_j = E[S_j], \quad \sigma_j^2 = Var[S_j], \quad j = 1, \ldots, m, \tag{3.6}$$

where ξ solves the equation $E[G_c] = const.$

Table of Lines Surplus

Similarly to the preceding result one has

$$1 - \pi_{T_j} = \min\left\{\xi\theta_r \frac{\mu_j}{\sigma_j^2}, 1\right\}, \quad j = 1, \ldots, m, \tag{3.7}$$

where ξ solves the equation $E[G_c] = const.$

Excess-of-Loss

For the collective model of risk theory (2.1) and an excess-of-loss reinsurance with deductible d, the mean and variance of the retained aggregate claims are given by

$$\begin{aligned} \mu_c &= \mu_c(d) = \lambda \cdot E[\min(X,d)] = \lambda \cdot (\mu_X - \pi_X(d)) \\ \sigma_c^2 &= \sigma_c^2(d) = \lambda \cdot E\left[\min(X,d)^2\right] = \lambda \cdot \left(\mu_X^2 + \sigma_X^2 - 2d\pi_X(d) - \pi_{2,X}(d)\right) \end{aligned} \tag{3.8}$$

with

$$\begin{aligned} &\mu_X = E[X], \quad \sigma_X^2 = Var[X], \quad \pi_X(d) = E[(X-d)_+], \\ &\pi_{2,X}(d) = E\left[(X-d)_+^2\right]. \end{aligned} \tag{3.9}$$

The (optimal) deductible is implicit solution of the equation
$(\theta_c - \theta_r)\mu + \theta_r \mu_c(d) = const.$

Stop-Loss

Similarly to excess-of-loss one has for an arbitrary aggregate claims random variable S:

$$\begin{aligned}
&\mu_c = \mu_c(L) = E[\min(S, L)] = \mu_S - \pi_S(L) \\
&\sigma_c^2 = \sigma_c^2(L) = E\Big[\min(S, L)^2\Big] - E[\min(S, L)]^2 \\
&= \mu_S^2 + \sigma_S^2 - 2L\pi_S(d) - \pi_{2,S}(L) - (\mu_S - \pi_S(L))^2 \\
&= \sigma_S^2 - 2(L - \mu_S + \pi_S(L)) - \sigma_S^2(L)
\end{aligned} \tag{3.10}$$

with

$$\begin{aligned}
&\mu_S = E[S], \quad \sigma_S^2 = Var[S], \quad \pi_S(L) = E\big[(S - L)_+\big], \\
&\pi_{2,S}(L) = E\Big[(S - L)_+^2\Big], \quad \sigma_S^2(L) = \pi_{2,S}(L) - \pi_S(L)^2.
\end{aligned} \tag{3.11}$$

The (optimal) deductible is implicit solution of the equation $(\theta_c - \theta_r)\mu + \theta_r\mu_c(L) = const.$

Variable Excess-of-Loss

In case a portfolio is divided into m segments, each with variable excess-of-loss deductible d_j, $j = 1, \ldots, m$, the mean and variance of the retained aggregate claims are given by

$$\begin{aligned}
\mu_c = \mu_c(d_1, \ldots, d_m) &= \sum_{j=1}^{m} \lambda_j \cdot E\big[\min(X_j, d_j)\big] = \sum_{j=1}^{m} \lambda_j \cdot \big(\mu_j - \pi_j(d_j)\big) \\
\sigma_c^2 = \sigma_c^2(d_1, \ldots, d_m) &= \sum_{j=1}^{m} \lambda_j \cdot E\Big[\min(X_j, d_j)^2\Big] \\
&= \sum_{j=1}^{m} \lambda_j \cdot \Big(\mu_j^2 + \sigma_j^2 - 2d_j\pi_j(d_j) - \pi_{2,j}(d_j)\Big)
\end{aligned} \tag{3.12}$$

with

$$\begin{aligned}
&\mu_j = E\big[X_j\big], \quad \sigma_j^2 = Var\big[X_j\big], \quad \pi_j(d_j) = E\Big[\big(X_j - d_j\big)_+\Big], \\
&\pi_{2,j}(d_j) = E\Big[\big(X_j - d_j\big)_+^2\Big].
\end{aligned} \tag{3.13}$$

To get the optimal deductibles according to the de Finetti criterion, we use the Lagrange function

$$\Phi(d_1, \ldots, d_m, \xi) = \sigma_c^2(d_1, \ldots, d_m) - \xi \cdot ((\theta_c - \theta_r)\mu + \theta_r\mu_c(d_1, \ldots, d_m) - E[G_c]) \tag{3.13}$$

The optimal parameters solve the first order conditions

$$\Phi_j = \frac{\partial \Phi}{\partial d_j} = 0, \; j = 1, \ldots, m, \qquad \frac{\partial \Phi}{\partial \xi} = 0. \tag{3.14}$$

Using the relationships

$$\frac{\partial \pi_{2,j}(d_j)}{\partial d_j} = -2\pi_j(d_j), \qquad \frac{\partial \pi_j(d_j)}{\partial d_j} = -[1 - F_j(d_j)], \tag{3.15}$$

where $F_j(d_j) = P(X_j \leq d_j)$ denotes the probability distribution of the random variable X_j, one obtains

$$\Phi_j = \lambda_j \bar{F}_j(d_j) \cdot (2d_j - \xi\theta_r) = 0, \; j = 1, \ldots, m, \tag{3.16}$$

hence the optimal deductibles

$$d_j = \frac{1}{2}\xi\theta_r = d, \; j = 1, \ldots, .m \tag{3.17}$$

are the same in each segment. The obtained unique deductible d is solution of the implicit equation

$$(\theta_c - \theta_r)\mu + \theta_r \cdot \sum_{j=1}^{m} \lambda_j \cdot (\mu_j - \pi_j(d)) = const. \tag{3.18}$$

Variable Stop-Loss

Similarly to the variable excess-of-loss one has for arbitrary $(S_1, \ldots, S_m)$ the expressions

$$\mu_c = \mu_c(L_1, \ldots, L_m) = \sum_{j=1}^{m} E[\min(S_j, L_j)] = \sum_{j=1}^{m} \left(\mu_{S_j} - \pi_{S_j}(L_j)\right)$$

$$\sigma_c^2 = \sigma_c^2(L_1, \ldots, L_m) = \sum_{j=1}^{m} E\left[\min(S_j, L_j)^2\right] - \mu_c(L_1, \ldots, L_m)^2 \tag{3.19}$$

$$= \sum_{j=1}^{m} \left(\mu_{S_j}^2 + \sigma_{S_j}^2 - 2L_j\pi_{S_j}(L_j) - \pi_{2,S_j}(L_j)\right) - \mu_c(L_1, \ldots, L_m)^2$$

Proceeding as above in the excess-of-loss case, one shows that the optimal priorities

$$L_j = \mu_c(L_1, \ldots, L_m) + \frac{1}{2}\xi\theta_r = L, \; j = 1, \ldots, .m \tag{3.20}$$

are the same in each segment. The obtained unique priority L is solution of the implicit equation

$$(\theta_c - \theta_r)\mu + \theta_r \cdot \sum_{j=1}^{m} \left(\mu_{S_j} - \pi_{S_j}(L)\right) = const. \tag{3.21}$$

To simplify calculations and for the ease of illustration, we use the following insurance risk models and mention that similar results can be obtained for any other risk models. For excess-of-loss, variable excess-of-loss, we use a *lognormal claim size* distribution with parameters m_j, s_j such that

$$\begin{aligned}
\mu_j - \pi_j(d_j) &= \mu_j \cdot \Phi\left(\frac{\ln(d_j) - m_j - s_j^2}{s_j}\right) + d_j \cdot \bar{\Phi}\left(\frac{\ln(d_j) - m_j}{s_j}\right) \\
\mu_j^2 + \sigma_j^2 - 2d_j\pi_j(d_j) - \pi_{2,j}(d_j) &= \left(\mu_j^2 + \sigma_j^2\right) \cdot \Phi\left(\frac{\ln(d_j) - m_j - 2s_j^2}{s_j}\right) \\
&+ d_j^2 \cdot \bar{\Phi}\left(\frac{\ln(d_j) - m_j}{s_j}\right)
\end{aligned} \tag{3.22}$$

For stop-loss and variable stop-loss, we use two different approximate aggregate claims distributions, a *lognormal model* with parameters μ_j, σ_j such that

$$\begin{aligned}
\mu_{S_j} - \pi_{S_j}(L_j) &= \mu_{S_j}\Phi\left(\frac{\ln(L_j) - \mu_j - \sigma_j^2}{\sigma_j}\right) + L_j\bar{\Phi}\left(\frac{\ln(L_j) - \mu_j}{\sigma_j}\right) \\
\mu_{S_j}^2 + \sigma_{S_j}^2 - -2L_j\pi_{S_j}(L_j) - \pi_{2,S_j}(L_j) &= \left(\mu_{S_j}^2 + \sigma_{S_j}^2\right)\Phi\left(\frac{\ln(L_j) - \mu_j - 2\sigma_j^2}{\sigma_j}\right) \\
&+ L_j^2\bar{\Phi}\left(\frac{\ln(L_j) - \mu_j}{\sigma_j}\right)
\end{aligned} \tag{3.23}$$

and a *Gamma model* with parameters $\alpha_j = \left(\mu_{S_j}/\sigma_{S_j}\right)^2$, $\beta_j = \alpha_j/\mu_{S_j}$, such that

$$\begin{aligned}
&\mu_{S_j} - \pi_{S_j}(L_j) = \mu_{S_j} \cdot \Gamma(\beta_j L_j; \alpha_j + 1) + L_j \cdot \bar{\Gamma}(\beta_j L_j; \alpha_j), \\
&\mu_{S_j}^2 + \sigma_{S_j}^2 - 2L_j\pi_{S_j}(L_j) - \pi_{2,S_j}(L_j) = \left(\mu_{S_j}^2 + \sigma_{S_j}^2\right) \cdot \Gamma(\beta_j L_j; \alpha_j + 2) + L_j^2 \cdot \bar{\Gamma}(\beta_j L_j; \alpha_j),
\end{aligned} \tag{3.24}$$

with $\Gamma(x;\alpha) = \frac{1}{\Gamma(\alpha)} \cdot \int_0^x t^{\alpha-1}e^{-t}dt$ the incomplete Gamma function and $\bar{\Gamma}(x;\alpha) = 1 - \Gamma(x;\alpha)$.

The use of a Gamma model of collective risk theory for sufficiently large portfolios can be motivated and justified (e. g. Hürlimann(2002)). Therefore, if the portfolio of the cedent is not too small, the calculation of RAC and RORAC of the retained risk is done in the unified framework of the Gamma model. For a confidence level α, mean retained risk μ_c and coefficient of variation $k_c = \sigma_c/\mu_c$ of the retained risk S_c, one has the formulas

$$VaR_\alpha[S_c] = \Gamma^{-1}\left(\alpha; \frac{1}{k_c^2}\right) \cdot k_c^2 \mu_c,$$

$$CVaR_\alpha[S_c] = VaR_\alpha[S_c] + \frac{1}{1-\alpha} E[(S_c - VaR_\alpha[S_c])_+]$$

$$= VaR_\alpha[S_c] + \frac{1}{1-\alpha}\left\{\mu_c \cdot \bar{\Gamma}\left(\frac{VaR_\alpha[S_c]}{k_c^2\mu_c}; 1 + \frac{1}{k_c^2}\right) - VaR_\alpha[S_c] \cdot \bar{\Gamma}\left(\frac{VaR_\alpha[S_c]}{k_c^2\mu_c}; \frac{1}{k_c^2}\right)\right\}. \tag{3.25}$$

4. Comparison Results

For concrete calculations and comparison of optimal reinsurance we consider a fictitious line of business divided into $m = 4$ segments, whose characteristics are summarized in the following Table 4.1. The loading factor of the cedent is first set at $\theta_c = 10\%$.

	Line of Business				Total
segment	**1**	**2**	**3**	**4**	**LoB**
Input Data					
Mean Claim Number	100	200	300	400	1000
Mean Claim Size	20	15	10	5	10
StDev Claim Size	200	120	100	40	102.7
Risk LoB					
Mean Aggr Claims	2000	3000	3000	2000	10000
Variance Aggr Claims	4040000	2925000	3030000	650000	10645000
St. Dev. Aggr Claims	2010.0	1710.3	1740.7	806.2	3262.7
Coeff. of Variation	1.005	0.570	0.580	0.403	**0.326**
VaR(99 %)	9256.6	8322.3	8439.2	4334.2	19106.8
CVaR(99 %)	11273.5	9531.0	9681.3	4813.8	20882.7
Insurance Premium	2200	3300	3300	2200	11000
ReinsurancPremium	0	0	0	0	0
RORAC	2.2 %	4.8 %	4.7 %	7.7 %	**10.12 %**
Expected Retaine Profit	200.0	300.0	300.0	200.0	**1000.0**

Table 4.1: Portfolio data and results without reinsurance, $\theta_c = 10\%$

Our comparison is based on a constant expected retained profit $E[G_c] = 700$ and varying loading factors $\theta_r \in \{7.5\,\%, 8.57\,\%, 10\,\%, 12\,\%, 15\,\%, 20\,\%, 30\,\%, 60\,\%\}$. With the assumption $\theta_c = 10\,\%$ this yields varying expected retained aggregate claims. The percentages of the latter to the expected aggregate claims lie in the set $\{60\,\%, 65\,\%, 70\,\%, 75\,\%, 80\,\%, 85\,\%, 90\,\%, 95\,\%\}$. Under these assumptions, the optimal retentions are determined according to the de Finetti criterion as described in Section 3 and listed at the end of the Section in Table 4.6. The corresponding values of the coefficient of variation are displayed in Table 4.2 and the values of RORAC in Table 4.3.

	Expected retained aggregate claims							
	60 %	**65 %**	**70 %**	**75 %**	**80 %**	**85 %**	**90 %**	**95 %**
Proportional								
quota share	0.3263	0.3263	0.3263	0.3263	0.3263	0.3263	0.3263	0.3263
surplus	0.3288	0.3284	0.3281	0.3279	0.3277	0.3275	0.3274	0.3272
variable quota share	**0.2933**	**0.2933**	**0.2933**	**0.2970**	**0.3006**	**0.3039**	**0.3070**	**0.3136**
table of lines surplus	0.2952	0.2949	0.2945	0.2981	0.3015	0.3047	0.3076	0.3138
Non-Proportional								
excess-of-loss	0.0792	0.0857	0.0931	0.1021	0.1130	0.1271	0.1467	0.1788
stop-loss model 1	**0.0449**	**0.0613**	**0.0807**	0.1034	0.1297	0.1605	0.1975	0.2447
stop-loss model 2	**0.0640**	**0.0810**	0.1004	0.1225	0.1476	0.1766	0.2110	0.2544
variable excess-of-loss	0.0785	0.0849	**0.0924**	**0.1012**	**0.1121**	**0.1261**	**0.1456**	**0.1775**
variable stop-loss model 1	0.1050	0.1184	0.1333	0.1500	0.1688	0.1906	0.2171	0.2524
variable stop-loss model 2	0.1404	0.1531	0.1670	0.1823	0.1994	0.2188	0.2420	0.2718

Table 4.2: minimum coefficient of variation for de Finetti's criterion

	Reinsurance loading factor							
	7.5 %	8.57 %	10 %	12 %	15 %	20 %	30 %	60 %
Proportional								
quota share	12.01 %	10.98 %	10.12 %	9.38 %	8.74 %	8.19 %	7.70 %	7.26 %
surplus	11.88 %	10.89 %	10.04 %	9.32 %	8.69 %	8.15 %	7.66 %	7.25 %
variable quota share	**13.85 %**	**12.65 %**	**11.64 %**	**10.61 %**	**9.73 %**	**8.98 %**	**8.32 %**	**7.64 %**
table of lines surplus	13.74 %	12.56 %	11.58 %	10.56 %	9.69 %	8.94 %	8.31 %	7.63 %
Non-Proportional								
excess-of-loss	108.62 %	79.32 %	60.17 %	46.62 %	36.45 %	28.42 %	21.74 %	15.71 %
stop-loss model 1	**1636.51 %**	**169.56 %**	**77.71 %**	45.68 %	29.87 %	20.66 %	14.70 %	10.50 %
stop-loss model 2	**187.27 %**	**88.47 %**	53.07 %	35.35 %	24.93 %	18.19 %	13.49 %	9.99 %
variable excess-of-loss	110.68 %	80.61 %	**61.05 %**	**47.24 %**	**36.90 %**	**28.74 %**	**21.97 %**	**15.86 %**
variable stop-loss model 1	62.70 %	45.45 %	34.23 %	26.45 %	20.77 %	16.45 %	13.01 %	10.09 %
variable stop-loss model 2	39.10 %	30.89 %	24.84 %	20.24 %	16.64 %	13.73 %	11.31 %	9.18 %

Table 4.3: maximum RORAC for de Finetti's criterion, $\theta_c = 10\%$

Besides the main facts a) and b) mentioned in the introduction, additional observations are made:

a) For *proportional reinsurance* the minimum coefficient of variation and the maximum RORAC are here attained for the *variable quota share*. As shown recently by Glineur and Walhin(2006) this is however not a general property of proportional reinsurance.

b) Appendix A shows that for a normal distribution quota share reduces the loss probability if and only if $\theta_r \leq \theta_c$. Table 4.3 shows that quota share increases RORAC if and only if $\theta_r \leq \theta_c$. However, the last property does not hold for the "optimal" variable quota share: with $\theta_r = 12\% > \theta_c = 10\%$ RORAC is higher than without reinsurance (10.61 % compared to 10.12 %).

c) For non-*proportional reinsurance* the minimum coefficient of variation and the maximum RORAC are attained either for *stop-loss* or (*variable*) *excess-of-loss*. For $\theta_r \leq \theta_c$ the stop-loss model 1 with lognormal retained aggregate claims is more efficient than (variable) excess-of-loss while for $\theta_r \geq \theta_c$ the stop-loss model 2 with Gamma retained aggregate claims is less efficient than (variable) excess-of-loss. However, in view of the Appendix, the considered non-proportional contracts are likely to be available on the reinsurance market only if $\theta_r \geq \theta_c$. This means that (variable) excess-of-loss is more efficient than stop-loss for the Gamma model and contradicts the traditional belief as already mentioned in the introduction.

d) There is a small region of parameters $\theta_r \geq \theta_c$, where the stop-loss model 1 is more efficient than the (variable) excess-of-loss. This example illustrates clearly the fact that optimality of reinsurance also depends on the chosen insurance risk models. Therefore the choice of optimal reinsurance is subject to "model risk".

e) The optimal variable excess-of-loss with constant deductibles is only slightly more efficient than excess-of-loss and the difference in deductibles is rather insignificant.

f) The variable stop-loss is always significantly less efficient than stop-loss and cannot be recommended as optimal non-proportional reinsurance contract. Even more, for the highest loading factors it produces a RORAC below the level without reinsurance and should be avoided.

To get more insight into the strong dependence of optimal non-proportional reinsurance upon the loading factors of the cedent and reinsurer, let us analyze the situation for $\theta_c = 8\,\%$ and varying $\theta_r \in \{10\,\%, 11.43\,\%, 13.33\,\%, 16\,\%, 20\,\%, 26.67\,\%, 40\,\%, 80\,\%\}$. Portfolio data is summarized in Table 4.4. The results on the minimum coefficient of variation and optimal retentions, which only depend on the proportion of retained aggregate claims, coincide with Table 4.2 and Table 4.6 while the maximum RORAC, which depends on the choice of θ_r, is found in Table 4.5. For the chosen θ_r values, no single proportional reinsurance improves the original RORAC value of 7.93 % without reinsurance and proportional reinsurance should be avoided. The comparative static conclusions for non-proportional reinsurance are identical to those made in case $\theta_c = 10\%$.

segment	1	Line of Business 2	3	4	Total LoB
Input Data					
Mean Claim Number	100	200	300	400	1000
Mean Claim Size	20	15	10	5	10
StDev Claim Size	200	120	100	40	102.7
Risk LoB					
Mean Aggr Claims	2000	3000	3000	2000	10000
Variance Aggr Claims	4040000	2925000	3030000	650000	10645000
StDev Aggr Claims	2010.0	1710.3	1740.7	806.2	3262.7
Coeff. of Variation	1.005	0.570	0.580	0.403	**0.326**
VaR(99 %)	9256.6	8322.3	8439.2	4334.2	19106.8
CVaR(99 %)	11273.5	9531.0	9681.3	4813.8	20882.7
Insurance Premium	2160	3240	3240	2160	10800
Reinsurance Premium	0	0	0	0	0
RORAC	1.8 %	3.8 %	3.7 %	6.0 %	**7.93 %**
Expected Gain Insurer	160.0	240.0	240.0	160.0	**800.0**

Table 4.4: Portfolio data and results without reinsurance, $\theta_c = 8\%$

	Reinsurance loading factor 10 %	11.43 %	13.33 %	16 %	20 %	26.67 %	40 %	80 %
Proportional								
quota share	6.53 %	5.99 %	5.54 %	5.15 %	4.82 %	4.52 %	4.26 %	4.02 %
surplus	6.46 %	5.94 %	5.50 %	5.12 %	4.79 %	4.50 %	4.24 %	4.03 %
variable quota share	**7.47 %**	**6.86 %**	**6.34 %**	**5.80 %**	**5.34 %**	**4.94 %**	**4.59 %**	**4.23 %**
table of lines surplus	7.41 %	6.81 %	6.30 %	5.77 %	5.32 %	4.92 %	4.59 %	4.22 %
Non-Proportional								
excess-of-loss	42.35 %	33.83 %	27.33 %	22.20 %	18.01 %	14.48 %	11.36 %	8.41 %
stop-loss model 1	**116.69 %**	**56.11 %**	**33.31 %**	21.83 %	15.13 %	10.85 %	7.90 %	5.74 %
stop-loss model 2	**59.37 %**	**36.66 %**	24.71 %	17.54 %	12.87 %	9.64 %	7.29 %	5.47 %
variable excess-of-loss	42.90 %	34.24 %	**27.65 %**	**22.45 %**	**18.21 %**	**14.62 %**	**11.47 %**	**8.49 %**
variable stop-loss model 1	28.24 %	21.74 %	17.06 %	13.58 %	10.90 %	8.78 %	7.04 %	5.53 %
variable stop-loss model 2	19.14 %	15.59 %	12.83 %	10.64 %	8.87 %	7.41 %	6.16 %	5.05 %

Table 4.5: maximum RORAC for de Finetti's criterion, $\theta_c = 8\%$

	Expected retained aggregate claims							
	60 %	65 %	70 %	75 %	80 %	85 %	90 %	95 %
Proportional								
quota share	60.0 %	65.0 %	70.0 %	75.0 %	80.0 %	85.0 %	90.0 %	95.0 %
surplus								
mean cession	60.0 %	65.0 %	70.0 %	75.0 %	80.0 %	85.0 %	90.0 %	95.0 %
secon moment cession	36.6 %	42.8 %	49.6 %	56.8 %	64.6 %	72.8 %	81.6 %	90.8 %
variable quota share								
segment 1	34.3 %	37.2 %	40.1 %	44.1 %	48.1 %	52.1 %	56.1 %	75.0 %
segment 2	60.5 %	65.6 %	70.6 %	77.7 %	84.7 %	91.8 %	98.8 %	100.0 %
segment 3	59.5 %	64.4 %	69.4 %	76.3 %	83.2 %	90.2 %	97.1 %	100.0 %
segment 4	85.6 %	92.8 %	99.9 %	100.0 %	100.0 %	100.0 %	100.0 %	100.0 %
table of lines surplus								
seg. 1/mean cession	34.3 %	37.2 %	40.1 %	44.1 %	48.1 %	52.1 %	56.1 %	75.0 %
seg. 1/second moment	12.0 %	14.1 %	16.3 %	19.7 %	23.3 %	27.4 %	31.7 %	56.5 %
seg. 2/mean cession	60.5 %	65.6 %	70.6 %	77.7 %	84.7 %	91.8 %	98.8 %	100.0 %
seg. 2/second moment	37.0 %	43.4 %	50.2 %	60.7 %	72.1 %	84.6 %	97.9 %	100.0 %
seg. 3/mean cession	59.5 %	64.4 %	69.4 %	76.3 %	83.2 %	90.2 %	97.1 %	100.0 %
seg. 3/second moment	35.9 %	42.0 %	48.7 %	58.7 %	69.8 %	81.8 %	94.7 %	100.0 %
seg. 4/mean cession	85.6 %	92.8 %	99.9 %	100.0 %	100.0 %	100.0 %	100.0 %	100.0 %
seg. 4/second moment	73.9 %	86.7 %	99.8 %	100.0 %	100.0 %	100.0 %	100.0 %	100.0 %
Non-Proportional								
excess-of-loss	72.0	97.8	134.9	190.5	279.6	436.5	762.5	1735.9
stop-loss model 1	6060.5	6617.2	7211.7	7863.7	8605.4	9495.6	10663.5	12515.9
stop-loss model 2	6097.7	6667.1	7274.0	7936.4	8682.8	9565.7	10698.5	12429.6
variable excess-of-loss	70.8	96.1	132.6	187.5	275.3	430.0	752.3	1717.5
variable stop-loss model 1	1697.3	1903.4	2140.5	2422.2	2771.6	3233.2	3910.4	5154.8
variable stop-loss model 2	1778.4	1994.9	2241.2	2529.7	2880.4	3330.2	3960.1	5023.6

Table 4.6: optimal retentions with de Finetti's criterion

Appendix: Inequalities for reinsurance loading factors

A. *Inequality for the loading factor of a quota share reinsurance*

A possible goal of reinsurance is reduction of the probability of technical ruin:

$$\Pr(S_c > P_c) \leq \Pr(S > P) \tag{A.1}$$

In case the aggregate claims random variables S_c, S can be well approximated by normal distributions, the condition (A.1) is equivalent with

$$\Phi((P-\mu)/\sigma) \leq \Phi((P_c-\mu_c)/\sigma_c) \quad \Leftrightarrow \quad P/\sigma \leq P_c/\sigma_c \tag{A.2}$$

with $\Phi(x)$ the standard normal distribution. Now, for a quota share reinsurance one has $S_c = qS$, hence $\sigma_c = q\sigma$. Inserting $P_c = P - P_r$, $P = (1+\theta_c)\mu$, $P_r = (1+\theta_r)(1-q)\mu$ into (A.2) one sees that (A.2) is fulfilled if and only if

$$\theta_r \leq \theta_c. \tag{A.3}$$

This inequality means that the reinsurer covers quota share reinsurance at a lower loading factor than the cedent.

B. *Inequality for the loading factor of excess-of-loss and stop-loss reinsurance*

The attention is restricted to stop-loss reinsurance but the arguments also hold for excess-of-loss. We derive a lower bound for the premium $P_r = (1+\theta_r) \cdot E\left[(S-L)_+\right]$, which implies a lower bound for the loading factor. We follow the stochastic dominance approach, which has been used to bound financial option prices (Lévy(1985), Henin and Pistre(1997), and the references therein). Given S with premium $P[S]$, consider the loss ratio $L(S) = \frac{S}{P[S]}$, which describes the cost per unit of money. Recall the notions of stochastic dominance and stop-loss order. The loss ratio $L(X)$ precedes $L(Y)$ in *stochastic dominance order*, written $L(X) \leq_{st} L(Y)$, provided the corresponding distribution functions satisfy the inequality $F_{L(X)}(x) \geq F_{L(Y)}(x)$ for all x in the common support of $L(X)$ and $L(Y)$. The loss ratio $L(X)$ precedes $L(Y)$ in *stop-loss order*, written $L(X) \leq_{sl} L(Y)$, provided the stop-loss transforms satisfy the inequality $\pi_{L(X)}(x) \leq \pi_{L(Y)}(x)$ for all x. We state the *hypothesis*, which leads to the desired bound.

(H) There exists no relation of stochastic dominance or stop-loss order between the loss ratios of two different (re)insurance strategies.

Under the assumption of homogeneous premium principles, the cost of two loss ratios is identical, namely one unit of money. The made hypothesis means that neither strategy can precede the other in stochastic dominance or stop-loss order. In other words the cost for the two strategies should differ. Consider the following two (re)insurance strategies:

(S1) Sell an insurance contract at price P to cover the claim S with loss ratio $L_1 = S/P$.

(S2) Sell a stop-loss reinsurance contract at price P_r to cover the claim $(S-L)_+$ with loss ratio $L_2 = (S-L)_+/P_r$.

The distribution functions of the considered loss ratios are given by

$$F_1(x) = F_X(P \cdot x), \quad x \geq 0 \tag{B.1}$$

$$F_2(x) = F_X(P_r \cdot (L + x)), \quad x \geq 0. \tag{B.2}$$

Since $P_r < P$ it is not difficult to see that

$$\begin{aligned} F_1(x) &< F_2(x), \quad x < c, \\ F_2(x) &> F_1(x), \quad x > c, \end{aligned} \tag{B.3}$$

where $c = P_r L/(P - P_r)$ satisfies $F_1(c) = F_2(c)$. This shows that the loss ratios L_1 and L_2 satisfy the once-crossing condition. In this situation, one knows that $L_1 \leq_{sl} L_2$ if and only if the means satisfy $E[L_1] \leq E[L_2]$. But, to prevent stop-loss order in accordance with the hypothesis (H), one must have $E[L_1] > E[L_2]$, that is

$$P_r > P \cdot E\big[(S-L)_+\big]/E[S]. \tag{B.4}$$

Under the expected value principle one has $P = (1+\theta) \cdot E[S]$, $P_r = (1+\theta_r) \cdot E\big[(S-L)_+\big]$. This implies that the inequality (B.4) is equivalent with the condition

$$\theta_r > \theta_c. \tag{B.5}$$

This inequality means that the reinsurer covers stop-loss reinsurance at a higher loading factor than the cedent.

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Zusammenfassung

Die grundlegenden Formen der Rückversicherung, namentlich die Quoten, Summenexzedenten, Excess-of-loss und Stop-loss Verträge und deren Varianten werden betrachtet. Die Optimalität der Rückversicherung bezüglich des Kriteriums von de Finetti und des RORAC Kriteriums unter Anwendung des Erwartungswertprinzips als Prämienkalkulationsprinzip wird analysiert. Es werden hinreichende Bedingungen hergeleitet, welche einen konstanten erwarteten Überschuss des Erstversicherers ergeben. Numerische Beispiele suggerieren folgende Aussagen. Die nicht-proportionale Rückversicherung ist, für einen festen erwarteten Rückbehalt des Gesamtschadens, effizienter als die proportionale Rückversicherung (gemessen an der Reduktion im Variationskoeffizient des Gesamtschadenrückbehalts). Zudem kann ein (variabler) Excess-of-loss, für einen höheren festen erwarteten Rückbehalt des Gesamtschadens, effizienter als einen Stop-loss sein, was den traditionellen Glauben widerspricht, dass Stop-loss eine optimale Rückversicherungsform ist. Dieselbe Vergleichseigenschaften gelten falls, für einen konstanten erwarteten Rückbehalt des Gesamtschadens und einen festen Zuschlagsfaktor der Rückversicherungsprämie, das RORAC Risikomass für ein approximatives Gamma verteiltes Modell des Gesamtschadenrückbehalts maximiert wird.

Résumé

Les formes de base de la réassurance, à savoir quote-part, excédent de pleins, excess-of-loss et stop-loss et leurs variantes sont considérées. Nous analysons l'optimalité de la réassurance par rapport au critère de Finetti et au critère RORAC en utilisant le principe de l'espérance mathématique comme principe de calcul des primes. On obtient des conditions suffisantes, qui produisent un profit de rétention espéré constant. Les exemples numériques suggèrent les faits suivants. La réassurance non-proportionelle est, pour une rétention espérée constante des sinistres totaux, plus efficace que la réassurance proportionelle (mesurée à la réduction du coefficient de variation des sinistres totaux retenus). De plus, un excess-of-loss (variable) peut, pour une rétention espérée constante suffisament élevée des sinistres totaux, être plus efficace qu'un stop-loss, ce qui contredit l'opinion traditionelle, qui affirme que stop-loss est une forme optimale de réassurance. Les mêmes propriétés restent valables lorsque la mesure RORAC est rendue maximale avec une rétention espérée constante des sinistres totaux et un facteur de surcharge des primes de réassurance constant pour un modèle des sinistres totaux retenus ayant une fonction de répartition approximativement Gamma distribuée.

Summary

The basic forms of reinsurance, namely quota share, surplus, excess-of-loss, stop-loss and some variants of them are considered. Optimality of reinsurance with respect to the de Finetti and RORAC criteria using the expected value principle for premium calculation is analyzed. Sufficient conditions which ensure a constant expected retained profit are derived. The analysis of numerical examples suggests the following statements. For fixed expected retained aggregate claims, non-proportional reinsurance is more efficient than proportional reinsurance (measured in terms of the reduction in the coefficient of variation of the retained aggregate claims). Moreover, for higher fixed expected retained aggregate claims (variable) excess-of-loss may be more efficient than stop-loss, which contradicts the traditional belief that stop-loss reinsurance is an optimal form. For constant expected retained profit and fixed reinsurance loading factor, the same comparative properties hold when maximizing the RORAC measure within an approximate Gamma distributed model of the retained aggregate claims.

Key words

optimal reinsurance, quota share, surplus, excess-of-loss, stop-loss, de Finetti criterion, RORAC criterion, ordering of risks