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Autor: Morrison, L.V. / Stephenson, F.R.

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Eclipses – a tool to measure the Earth's rotation

L. V. MORRISON, F. R. STEPHENSON

Ancient and medieval records of solar and lunar eclipses provide valuable information about the rotation of the Earth. Using these seemingly crude observations, variations in the Earth's rotation can be traced back in some detail over the past 2500 years.

Empress of Kao-tzu, 7th year, first month, day chi-ch'ou, the last day of the month. The Sun was eclipsed; it was total; it was 9 deg in [the lunar lodge] Ying-shih, which represents the interior of the Palace chambers. At that time the [DOWAGER] Empress of Kao-[tzu] was upset by it and said, «This is on my account.» The next year it was fulfilled.

The Empress DOWAGER died nearly 18 months afterwards, on 18 August in BC 180, and the eclipse is identified as that of BC 181 March 4. This account of the eclipse is taken from a history of the Former Han Dynasty (Han-shu) which was compiled in AD 58-76. The «Palace» in the extract was sited in the capital at that time, Ch'ang-an. The interpretation of the eclipse as a portent of doom for the emperor or his household was typical throughout Chinese history. Whilst the eclipse may have been portentous for the Empress, it is propitious for geophysics, because the fact that the path of the total eclipse of BC 181 March 4 passed over Ch'ang-an, fixes the rotational angle of the Earth at that epoch to within the width of the narrow track, as shown in Figure 1.

Figure 1(a) shows the path of totality passing over Chang'an (modern Xi'an), as described in the dynastic history. However, when we calculate the position of the Earth on the assumption of uniform rotation on its axis between 181 BC and the present, we find that the path of totality is as shown in Figure 1(b). The rotational displacement between Figure 1(a) and 1(b) is about 3.4 hours. This is, therefore, the cumulative deviation in rotation angle due to variations in the Earth's rate of rotation between 181 BC and the present. With many such observations, and also timings of eclipses, it is possible to trace the behaviour of the Earth's rotation in the past. One of us has compiled numerous historical observations of eclipses (STEPHENSON, 1997) for this purpose. Before we discuss the provenance and analysis of these observations, we outline the main factors affecting the long-term behaviour of the Earth's rotation.

Tidal friction and the rotation of the Earth

Tidal friction is the predominant long-term mechanism acting to change the rate of rotation of the Earth. The Earth's rotation decelerates under the action of the torque exerted by the Moon on what may be visualised as the Earth's two tidal bulges which are displaced from the Earth-Moon line because of the anelastic response of the Earth. Solar tides also contribute a smaller component to the deceleration. The angular momentum lost by the deceleration of the Earth due to lunar tidal friction is transferred via the tidal torque to the orbit of the Moon. As a consequence of Kepler's Third Law, the Moon's orbit expands (by about 3.7 cm per year) and the Moon decelerates in its angular motion.

The transfer of angular momentum occurs mainly in the oceans and seas, probably by side pressure on the coasts and oceanic mountain ridges. It is reasonable to suppose that on the millennial time-scale there is negligible change in the transfer of angular momentum in the highly stable deep oceans. There are also grounds for believing that even the shallow sea contribution has remained almost constant over the past few millennia (PIRAZZOLI, 1991; MORNER, 1971). There is thus no good reason for supposing that the tidal acceleration of the Earth has varied significantly over the last 2500 years.

Tidal acceleration of the Moon's orbital motion

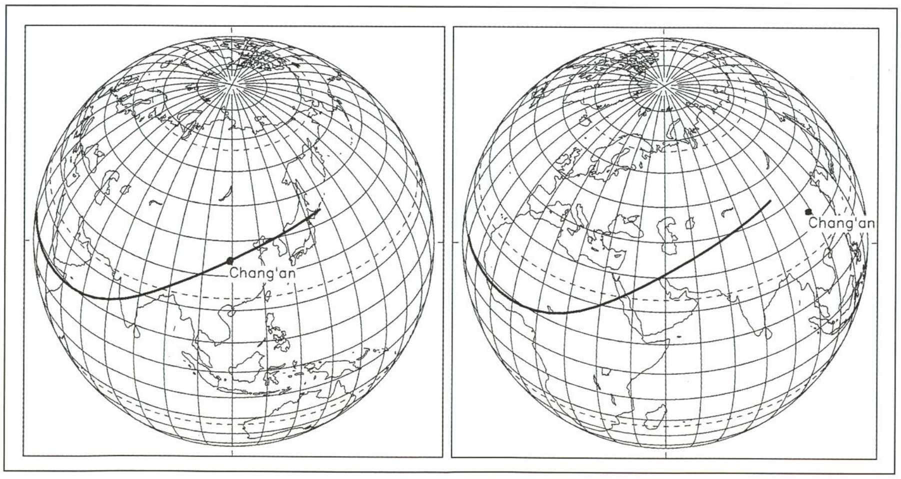
The angular acceleration of the Moon over the past 250 years has been measured by analysing observations of its motion relative to stars, mainly by timing occultations. This method assumes the existence of a uniform timescale against which to make the observations. The rotation of the Earth itself is decelerating, so the timescale which is derived from the period of rotation of the Earth cannot be used for this purpose. An independent timescale based on the regularity of planetary motion has to be used (the atomic time-scale was not introduced until 1955). This method was used by MORRISON and WARD (1975) to derive a value of $-26\pm2''/cy^2$ for the tidal acceleration of the Moon.

This result has been confirmed by the more direct method of laser ranging to the corner-cube reflectors placed on the Moon. From the analysis of 18 years of data WILLIAMS et al. (1992) obtained $-25.9\pm0.5''/cy^2$. In our work we have adopted a value of $-26''/cy^2$ exactly, with an uncertainty of $\pm0.5''/cy^2$.

Tidal acceleration of the Earth's spin

Knowing the tidal acceleration of the Moon and the corresponding expansion of its orbit allows us to calculate the gain in angular momentum in the Moon's orbit. By the conservation of angular momentum in the Earth-Moon system, the Earth loses in spin what the Moon gains in its orbit. Allowing for a contribution from solar tides (see CHRISTODOULIDIS et al., 1988), the tidal acceleration of the Earth comes out as $-6.15\pm0.37 \times 10^{-22}$ rad/s^2. Another way of deriving the tidal acceleration of the Earth is to measure the ef-

Figure 1. (a) Observed path of the total solar eclipse of 181 BC passing over Chang'an, and (b) the computed path based on the assumption that the length of day (l.o.d.) has been constant. The difference in longitude is 51 degrees, which is equivalent to 3.4 hours.



fect on the orbits of artificial satellites of the tidal potential. By this method CHRISTODOULIDIS et al. (1988) found $-5.98\pm0.22 \times 10^{-22}$ rad/s^2. This is in good agreement with the value above which is deduced from the Moon's acceleration measured over a time-span of 250 years. We adopt the coefficient –6.15.

Change in the length of the day

The length of the day (l.o.d.) is a convenient unit of measure for the rate of rotation of the Earth. The standard unit of comparison is the day of 86400 s SI. A change of +1 millisecond (ms) in the l.o.d. is approximately –1 part in 10^8 of the Earth's rate of spin and is equal to -0.843×10^{-12} rad/s^2. A rate of change in the l.o.d. of +1 ms per century (cy) is -2.67×10^{-22} rad/s^2 which has a similar scale to the value of the tidal acceleration, $-6.15\pm0.37 \times 10^{-22}$ rad/s^2. So, we can conveniently express this acceleration as a rate of change in the l.o.d. of +2.3±0.1 ms/cy.

The mean length of the day was equal to the standard day of 86400s SI around the epoch AD 1820, so that epoch is adopted here as the zero point from which intervals of time are measured. If a is the Earth's rotational acceleration, the cumulative discrepancy in time of the Earth's clock is given by $\frac{1}{2} a t^2$. Converting from units of angular acceleration, rad/s^2, to s/cy^2, introduces the factor -13.7×10^{22}. Thus the cumulative discrepancy in time due to a tidal acceleration of $-6.15\pm0.37 \times 10^{-22}$ rad/s^2 is expressed in seconds by $(+42\pm2)\, t^2$, where t is the time in centuries from AD 1820.

Historical observations of eclipses give us the total discrepancy of the Earth's clock due to tides and other possible influences, such as changes in the moment of inertia or interchange of momentum between the mantle and core of the Earth. Whereas other methods exist for measuring the tidal component, eclipses are the only reliable way of measuring the non-tidal components in the pre-telescopic period, and therein lies their importance to geophysics.

Historical observations of eclipses of the Sun and Moon

Eclipses of the Sun and Moon are striking phenomena which have been observed with interest and often foreboding by most civilizations. Sometimes the date, time and place where the eclipses were observed were carefully recorded. Eclipses of the Sun and Moon can be timed with the unaided eye to within a minute or two without difficulty, and this is certainly good enough for our present purpose. However, this precision is not attained in historical observations, mainly because of the difficulty of measuring time with crude instruments. Nevertheless, we shall show that they are still more than adequate for our purpose.

Fortunately, in the particular case of a total solar eclipse, it is not necessary to know the time of day of when the eclipse occurred because the path of totality is narrow and this in itself fixes the position of the Earth, as illustrated in Figure 1. All we require to know is the date and place at which the eclipse was reported to have been total. We refer to these observations of total solar eclipses as untimed events because their utility is independent of timing and relies on the geometrical circumstances of the events. For most other events, we require the time of day, and we refer to these as timed events. The untimed and timed events give us two independent sets of data with which to investigate variations in the l.o.d. over the past 2500 years.

A necessary prerequisite to analysing eclipses is the possession of reliable gravitational theories of the motions of the Moon and the Sun (the Earth's motion reflected). Such theories are available today, including the important evolutionary change in the Moon's orbit due to its tidal interaction with the Earth.

Many reliable records of timed and untimed observations of eclipses were made in the following four civilizations: ancient Babylon; ancient and medieval China; ancient and medieval Europe; and the medieval Arab world. We give an outline of the sources of the observations and also discuss their quality. For greater details, the reader is referred to STEPHENSON (1997).

Babylon

At some time before 700 BC, Babylonian astronomers began to systematically predict and observe eclipses of both Sun and Moon and this practice continued until perhaps as late as the first century AD.

Original Babylonian eclipse observations inscribed on clay tablets survive in large numbers. Virtually all of the known records of this kind are now in the British Museum, having been recovered from the site of Babylon rather more than a century ago (SACHS, 1974; STEPHENSON and WALKER, 1985). Regrettably, most tablets are badly damaged and only about 10 per cent of the original material is known to be extant.

Day-to-day astronomical diaries provide the most original sources of observations. Photographs, transliterations and translations of all of the datable diaries from 652 BC (the earliest known example) down to 50 BC have been published (SACHS and HUNGER, 1988, 1989, 1996).

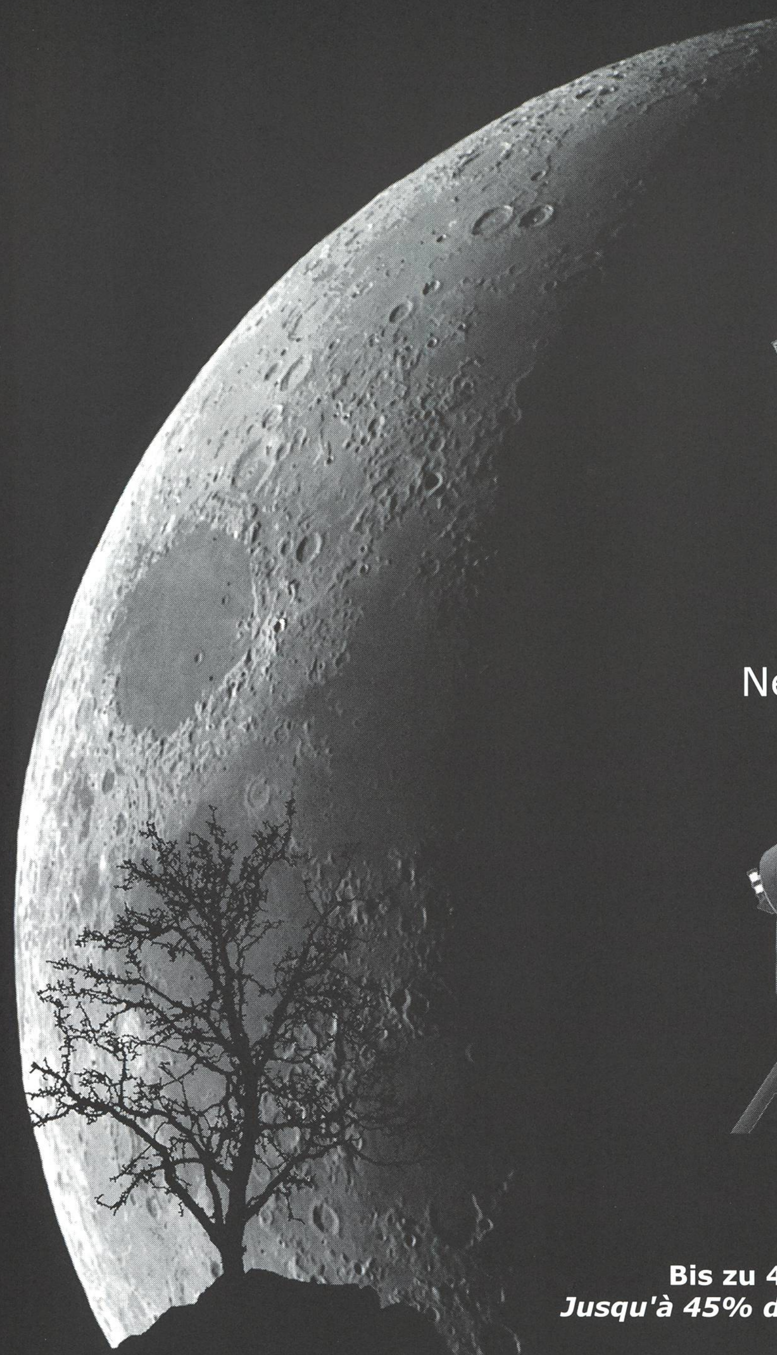
Almost all of the observations preserved in the astronomical diaries date from after 350 BC. Fortunately, compilations of eclipse records containing observations going back as far as about 700 BC are also extant. PETER HUBER (1973), lately of Bayreuth University, provided transliterations and translations of numerous solar and lunar eclipse records which he extracted from the available diaries, goal-year and eclipse texts.

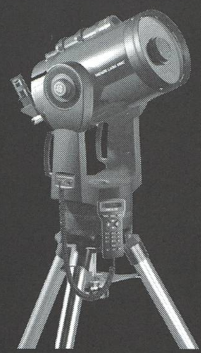
The Babylonian calendar was luni-solar, the first month of each year beginning around the time of the spring equinox. Months began with the first visibility of the crescent Moon, while the day commenced at sunset. Extensive studies of the Babylonian calendar have enabled dates between 626 BC and AD 75 to be converted accurately to our modern calendar (PARKER and DUBBERSTEIN, 1956). Experience has shown that high reliance can be placed on the Babylonian dates.

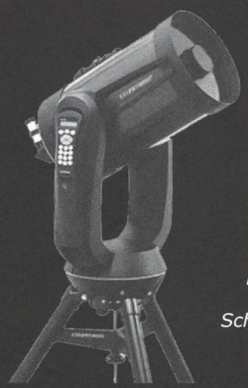
Throughout the period covered by the texts, Babylonian astronomers systematically timed the interval between the onset of an eclipse and sunrise or sunset (whichever was nearer). Probably some kind of clepsydra (water clock) was used for timing the various contacts. The standard unit of time adopted was the *uš*, being equal to 1/360 of the day and night and thus 4 min. Since this unit was the interval required for the celestial sphere to turn through 1 degree, it is customary to translate *uš* directly as time-degree (contracted to deg, here). By way of an example, here is a transliteration of an observation of a lunar eclipse taken from the tablet numbered BM 41536 in the British Museum:

[..] year 42, month XII 15, 1;30 (=90) deg after sunset [..] 25 deg duration of maximal phase. In 18 deg it became bright. West (wind). Went 2 cubits below γ Vir eclipsed. (trans. P. J. HUBER, 1973).

Some detective work is required to date the eclipse because the name of the king in whose reign it occurred is missing from the beginning of the damaged tablet, which is part of a table of lunar eclipses. It now records only two eclipses: in the 6th and 12th months of the 42nd year of the king's reign. Such a long reign could only refer to either NEBUCHADREZZAR II (42nd year = BC 563/2) or ARTAXERXES II (BC 363/2). However, the terminology is early, and in any







case there were only eclipses in the 3rd and 9th months of the appropriate year of ARTAXERXES II. Hence the former year must be intended. Calculation shows that when the eclipse occurred, the Moon would have been about 3 deg south of γ Vir, in close accord with the record. Since the interval between the first and second contacts in the text is not preserved, only the timing of the start of the eclipse can be utilised.

The principle by which the error in the Earth's clock is derived from this and similar timings is illustrated in Figure 2. The upper part of the Figure shows the position of Babylon at sunset, and the second its position as the Moon enters the umbra. The local time of sunset is known, and adding the elapsed time gives the observed time of the beginning of the eclipse, which corresponds to the rotational phase of the Earth. This is compared with the calculated time which is based on the assumption that the Earth's rate of rotation has been constant between the date of the eclipse and the present. The difference gives the *discrepancy in time* between the Earth's clock and a perfect clock keeping a constant rate. Each timed observation gives a discrete measure of the *discrepancy in time* for the epoch of the eclipse.

The predominant uncertainty in the observation is the measurement of the elapsed time using a water clock. However, there are circumstances when the eclipse begins very soon after sunset, and the error of the water clock is thereby reduced. These are accorded greater weight in the subsequent analysis. In the limiting case where the eclipse begins before sunset, the observation can be treated as an untimed event, relying on the geometry of situation to set limits to the possible rotational configuration of the Earth. The observations made relative to sunrise follow an analogous treatment.

China

Although Chinese records of solar eclipses commence around 700 BC, there was little interest in reporting lunar eclipses until about AD 400, possibly because they were regarded as less serious omens than their solar counterparts. However, from these respective dates, the sequence of recorded events continues almost uninterrupted down to recent times. Most of these accounts are very brief, giving no more than the date of occurrence, and are thus of negligible value for the present purpose. However, a small percentage of the observations of both solar and lunar eclipses contain important details.

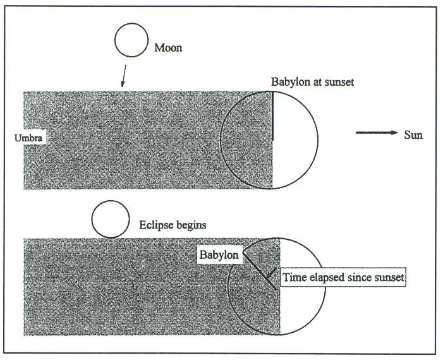


Figure 2. Schematic view of the time elapsed between sunset and the beginning of a lunar eclipse.

The principal sources of eclipse observations in Chinese history are the official dynastic histories. These have been reprinted many times and except for the last (Qing) dynasty, all of the original reports have long since perished. In most of these works, eclipse observations are mainly to be found in two sections: a special treatise devoted to astronomy (including astrology), and the imperial annals. Additionally, the calendar treatises of a few histories also pay special attention to timed eclipses. Eclipse observations cited in both the astronomical and calendrical treatises are probably nearly all derived from the records of the court astronomers, who maintained a regular watch for celestial phenomena of all kinds at the imperial observatory. Observations reported in the imperial annals are of more uncertain origin.

We have mainly confined our attention to eclipses cited in the treatises of the official dynastic histories. Some additional material has been extracted from chronicles and historical compendia such as the great *Wenxian Tungkao* (Comprehensive history of civilization) compiled by MA DUANLIN around AD 1280. Recently, Beijing Observatory (1988) assembled an extensive list of celestial observations of all kinds (including eclipses) preserved in Chinese history. This work has proved to be a valuable secondary source.

In expressing dates, years were numbered from the start of each reign-period. Like the Babylonian calendar, the Chinese calendar was luni-solar. The first month of each year began roughly midway between the winter solstice and the spring equinox. From very ancient times the Chinese also adopted a continuous 60-day cycle, independent of any astronomical parameter. This practice considerably facilitates date computation. Tables produced by various specialists (e.g. HSUEH CHUNG-SAN and OU-YANG I, 1956) enable all dates from the beginning of the Han Dynasty (202 BC) to be accurately converted to our modern calendar.

From at least AD 400, it was the practice of Chinese astronomers to measure times of solar eclipses to the nearest *ke* («mark»), equal to 1/100 of a day and night or 0.24 h. Previously, such times had usually been estimated to no better than the nearest double hour. At about the same date, lunar eclipses began to be timed to the nearest fifth of a *geng* («night watch»), but after about AD 1000, *ke* were preferred for all measurements. Although the units termed *ke* were of fixed length, the five night watches varied with the seasons. The standard timing device was a clepsydra, which was adjusted for seasonal variations in the lengths of the units when necessary (NEEDHAM et al. 1986). Such careful measurements were probably made almost exclusively at the capital, since few accurate instruments would be available in the provinces. Most recorded timings which are still preserved are restricted to two discrete periods: from AD 400 to 600 and again from AD 1000 to 1300. Presumably many measurements in the intervening centuries have gone missing.

An example of a Chinese record of a lunar eclipse timing is as follows: AD 596 Dec 11 (capital Daxing Zheng):

Kai-huang reign period, 16th year, 11th month, 16th day, yichou *[2].... Not until the first rod of the third watch was the Moon seen in the clouds above the direction* ping *(roughly SSE, azimuth approximately 165 deg). It was already about three fifteenths eclipsed and the loss began from the east side. Above the direction* ting *(roughly SSW, azimuth approximately 195 deg), the eclipse was total. Afterwards it reappeared from the SE side. Not until the third rod of the fourth watch was it restored to fullness; the Moon was then at the end of the direction* wei *(azimuth approximately 217.5 deg).* Sui-shu (chap. 17).

Europe.

Numerous untimed solar and lunar eclipses are recorded in the ancient Greek and Latin Classics, but in virtually every case either the date, place of observation or eclipse magnitude (or a combination of these factors) is uncertain. With a single exception, we have not used them for this reason. However, PTOLEMY lists a small number of ancient Greek timings of lunar eclipses in his *Almagest* (books IV and VI). Both date and place of observation are well established.

Medieval European reports of solar eclipses are often very reliable. From about AD 800 to 1500, chroniclers in towns and monasteries frequently noted the most striking celestial phenomena such as eclipses, comets, meteor showers and the *aurora borealis*. Dates in these works are usually accurate and since many chronicles were mainly concerned with local events, the place of observation can usually be taken as the town or monastery where the annalist lived. Descriptions of particularly large eclipses are often vivid and highly original; frequently it is reported that the chronicler himself witnessed the event. Accurate times are never given in annals, so that the lunar eclipse observations are of no value to us. However, many accounts of solar obscurations either carefully describe the complete disappearance of the Sun or affirm that a small part of the solar disk remained unobscured. More observations in these categories are preserved in medieval European annals than in any other early source.

A large number of medieval European chronicles have been published in their original language (which is usually Latin) by editors such as MURATORI (1723-) and PERTZ (1826-). About a century ago, CELORIA (1877a, 1877b) and GINZEL (1884a, 1884b, 1918) made extensive searches of the published chronicles for accounts of solar eclipses. They were able to uncover numerous records which they quoted in their original languages. NEWTON (1972), who gave valuable historical notes, provided translations of many of these records, though several of his quotations are incomplete. Wherever possible we have consulted the published chronicles and we have also made full use of the material compiled by CELORIA and GINZEL.

Most medieval chroniclers used the Julian Calendar, employing the terms Kalends, Nones and Ides. Seasonal hours (12 to the day and 12 to the night) were in common use, noon occurring at the 6th hour of the day. However, as noted above, times as reported in chronicles were only crudely estimated.

The following example indicates the quality of some of the observations reported in medieval European chronicles. AD 1267 May 25 (Constantinople).

At that time the Moon obscured the Sun when it was in the 4th part (degree) of Gemini, at the 3rd hour before midday on the 25th day of May in the year 6775 (Byzantine, i.e. A.D. 1267). It was a total eclipse of about 12 digits or points. Also, such darkness arose over the Earth at the time of mid-eclipse that many stars appeared. [NICEPHORAS GREGORAS Hist. Byzant., *Lib. IV, cap. 8; Migne (1865)*].

The most important eclipse observations in the centuries immediately preceding the invention of the telescope are by the Jesuit astronomer CHRISTOPHER CLAVIUS (1593). He observed a total eclipse in AD1560 and one which was virtually complete in 1567 and although no times were measured, he provided detailed descriptions of them (STEPHENSON, JONES & MORRISON, 1997).

Arab Dominions

Medieval Arab records of eclipses are mainly to be found in two quite distinct sources: chronicles and astronomical treatises, the latter being termed *Zij*. Chronicles cover much the same period as the town and monastic annals of Europe (roughly from AD 800 to 1500). Further, the untimed and essentially qualitative descriptions of eclipses and other celestial phenomena contained in these works have much in common with those of European origin. Unfortunately, relatively few Arabic chronicles appear to be preserved, so the number of extant reports of eclipses is correspondingly small. The few *Zij* which record eclipse observations cover only a relatively brief period – from around AD 800 to 1000. These compilations (notably the *Zij al-Kabir al-Hakimi* of IBN YUNUS, dedicated to Caliph AL-HAKIM), contain many measurements of the times of both solar and lunar eclipses.

Dates are normally expressed in terms of the Islamic lunar calendar. This assigns to every year 12 months, each of length 29 or 30 days. Hence the Islamic year contains only 354 or 355 days, with the result that the beginning of the year continually retrogrades relative to our modern calendar, making a full cycle of the seasons in about 33 years. Years on this scheme (designated AH) are numbered from the *Hijra*, the migration of Muhammad from Mecca to Medina in AD 622. Tables for the rapid conversion of Muslim dates to the Julian or Gregorian Calendar have been produced by FREEMAN-GRENVILLE (1977).

The lunar eclipse observations reported in Arab chronicles are probably too crude to be of value for the present purpose. On the other hand, several total solar eclipses are reported in graphic detail. Among these may be cited the following entry in the chronicle of IBN AL-JAWZI on a date corresponding to AD 1061 June 20:

(453 AH). *On Wednesday, when two nights remained to the completion of Jumada al-Aula (the 5th month), two hours after daybreak, the Sun was eclipsed totally. There was darkness and the birds fell whilst flying. The astrologers claimed that one-sixth of the Sun should have remained (uneclipsed) but nothing of it did so. The Sun reappeared after four hours and a fraction. The eclipse was not in the whole of the Sun in places other than Baghdad and its provinces.* (trans. S.S. SAID et al. 1989).

As well as giving the correct date, this account is quite definite with regard to the complete disappearance of the Sun and furthermore it clearly specifies the place of observation.

Rather than use a clepsydra, medieval Muslim astronomers preferred to measure eclipse times indirectly by determining altitudes using a quadrant or astrolabe, afterwards reducing their results to local time. These instruments were probably much more accurate than the crude timing devices of the period. The following example from the *Zij* of IBN YUNUS indicates the care with which Muslim astronomers often measured the times of eclipses:

AD 923 Jun 1 (Baghdad: lunar eclipse reported by ALI IBN AMAJUR AL-TURKI):

There was an eclipse of the Moon in (the month of) Safar in the year 311 of al-Hijrah.... The Moon rose at sunset (already) eclipsed by 1/4 or (a little) more of the digits of the diameter (i.e. 3 digits or a little more). The Moon was eclipsed by (a little) more than 9 digits of diameter. The middle of the eclipse was at 1 and 2/3 of equal hours of night (i.e. after sunset). The clearance of the eclipse was at 3 equal hours (after sunset) and (that was) when the altitude of (the star) al-ridf (Deneb: α Cyg) was 29;30 deg in the east... (trans. S.S. SAID and F. R. STEPHENSON, 1997).

Analysis of the eclipses: geophysical results

The two independent datasets of untimed and timed eclipses (see Table 1) have been analysed by us (STEPHENSON and MORRISON, 1995) for changes in the Earth's rotation over the past 2500 years. Here we describe the main points of that analysis, updated by the inclusion of more data which we have investigated over the past few years.

Untimed data

Each untimed total solar eclipse gives a range of values with sharp boundaries for the difference between

Table 1: Numbers of eclipse observations

Source	Untimed	Timed
Babylon	32	152
China	35	121
Arab	13	59
Europe	26	11

the predicted and observed longitudes of the path of totality. This difference is dependent on the width of the path and its angle relative to the equator, as can be seen from Figure 1. Differences in longitude, converted to units of time, are equivalent to the discrepancy in time between the standard clock and the Earth's clock. In the case of Figure 1, the range of possible values for the discrepancy in time lies between 3.28 and 3.53 hours. We refer to this as the solution space, and it is plotted as a vertical bar at epoch 181 BC in Figure 3. The results from other total solar eclipses are plotted similarly.

Where an eclipse was reported as being not quite total, the solution space lies on either side of that for totality. These are plotted as bars with arrowheads. One of these two sections of the solution space can sometimes be discarded because it is redundant. Even though the solution space is almost unbounded in the direction of the arrowhead, the other end is sharp and often produces an effective limit.

Observations that the Moon (or rarely the Sun) rose or set while eclipsed, produce similar types of solution space as total or near-total solar eclipses, but with wider boundaries. These are labelled «solar and lunar horizon obs.» in Figure 3.

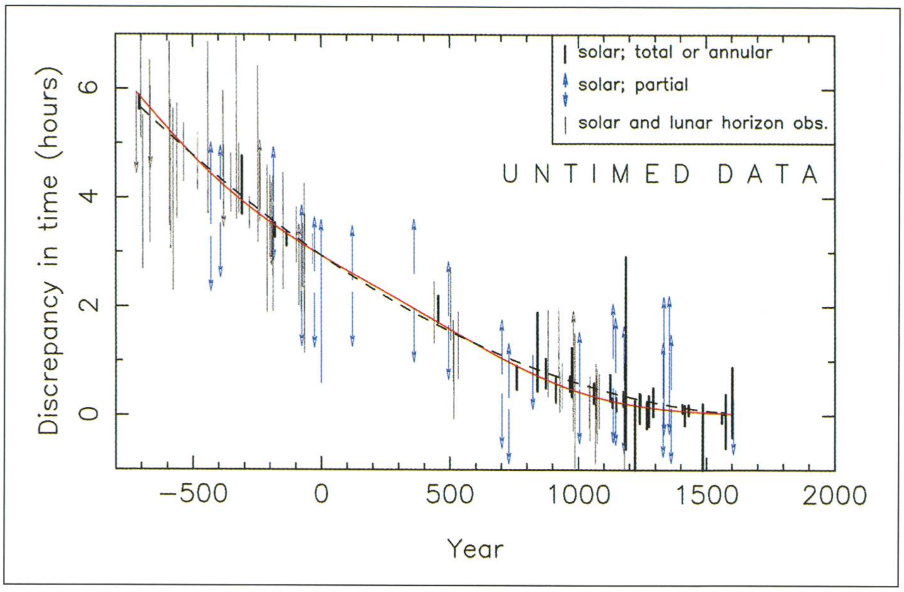


Figure 3. Plot of the difference between the computed and observed positions of eclipses from –719 to +1605. The dashed curve is the best-fitting parabola. The red curve was fitted by cubic splines to the data in Figures 3 and 4. Note that the vertical lines are not error bars, but solution space, anywhere in which the actual value is equally likely to lie. Arrowheads denote that the solution space extends several hours in that direction.

Timed data

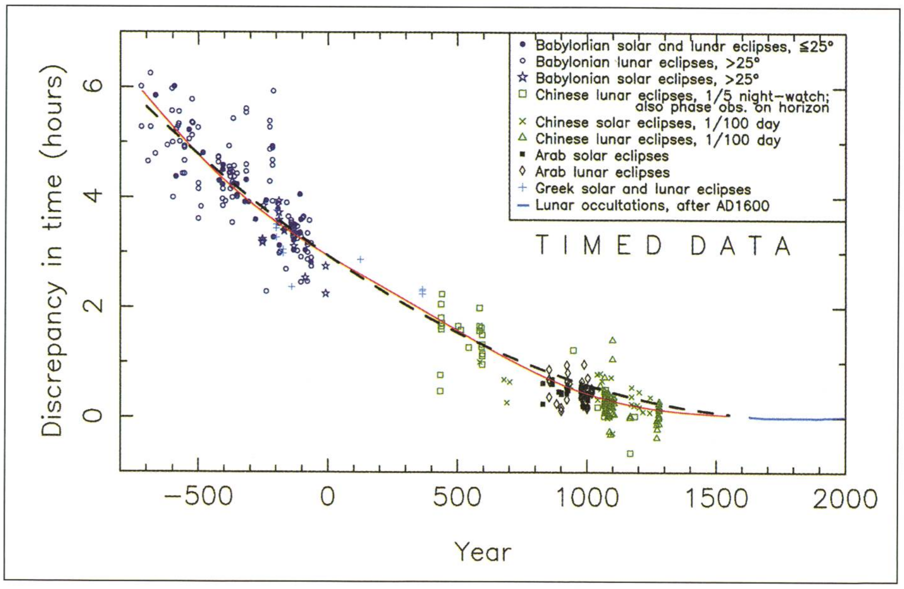
The discrepancies in time between the predicted and observed times of solar and lunar eclipses are plotted as discrete points in Figure 4. They are subject to considerable error, of course, and estimates of these can be made from the vertical scatter of the points. After the introduction of the telescope in the early part of the 17th century, the timing of thousands of occultations of stars by the Moon produces much higher resolution, and the discrete points lie close to a narrow, continuous curve. This is shown as a continuous blue line in Figure 4. The errors on this curve range from a few tens of seconds in the 17th century to less than 1 s by the 19th century, and are thus negligible on the scale of Figure 4.

Curve-fitting to the raw data

It is clear from Figures 3 and 4 that the data follow a very similar trend. The simplest model that one might propose to fit these data is a parabola with its apex at zero around +1820 which is the epoch at which the rate of Earth's clock is equal to the standard of comparison. A parabola would be the result of a combination of constant forces acting to decelerate the Earth's rotation. The best-fitting parabola is given by $+32\ t^2$ (seconds), where t is centuries from the epoch +1820. This is shown as a dashed curve in Figures 3 and 4.

Whilst in general this parabola is a good fit to most of the data, it does not satisfy the very reliable boundary conditions of several eclipses in Figure 3. The critical untimed observations and all the timed observations in the period +400 to +1600 are reproduced on an enlarged scale in Figure 5. The parabola does not satisfy the constraints imposed by the untimed total solar eclipses of +454, +761, +1133, +1147 (annular), +1221 and +1267, and the partial solar eclipses of +1178 and +1361. The historical accounts of these eclipses are all very reliable. If the parabola were correct, these total/annular eclipses would in fact have been partial at the places of observation, and in the case of the partial eclipses, the parabola passes through the solution space which could only be satisfied if the eclipses were total at the places of observation. However, the de-

Figure 4. Plot of the difference between the computed and observed times of eclipses from –720 to +1279. The dashed curve is the best-fitting parabola. The red curve was fitted by cubic splines to the data in Figures 3 and 4. The blue curve after AD 1600 was derived mainly from lunar occultations and its uncertainty is less than its width.



scriptions of these eclipses clearly precludes this interpretation. Here is a summary of the accounts of these discrepant eclipses:.

+ 454 China .. *it was total; all the constellations (i.e. lunar lodges) were brightly lit.*

+ 761 China .. *the large stars were all seen.*

+1133 Europe (several independent reports) .. *day was turned into night, very many stars were seen/.. the Sun, as if it did not exist, was entirely concealed/.. the Sun suddenly disappeared.. and stars also appeared in the sky.*

+1147 Europe (annular) .. *a circle of different colours and spinning rapidly.*

+1221 China .. *all the stars were therefore seen.. at that time we were on the southern bank of the [Kerulen] river.* (See STEPHENSON (1997) for a discussion of the observers relative to the course of the river.).

+1267 Europe .. *such darkness arose over the Earth at the time of mid-eclipse that many stars appeared.*

The appearance of many stars in these reports is a sure indication of totality. In the case of the annular eclipse of +1147, the description supports the view that the eclipse was central at the point of observation.

+1178 Europe .. *its disc began to be covered from the east (sic) until it was like a two- or three-day old moon.*

+1361 China .. *suddenly it lost its light.. it took the shape of a plantain leaf* (i.e. oval in shape).

These two eclipses were clearly not total at the places of observation.

We note that in the period +700 to +1400, the parabola lies above all the critical limits of the eclipses discussed here. Whilst, the location of some of the observers could be questioned, it is very unlikely that these independent accounts could result in such a systematic displacement. The other crucial factor in rejecting a simple parabola is the displacement of the timed data below the parabola between +800 and +1300 in Figure 5. The completely independent Arab (black) and Chinese (green) data clearly indicate a solution below the parabola (dashed curve) in Figure 5.

For these reasons, we rejectd a purely parabolic solution. Curve-fitting by cubic splines with knots at the epochs –200, +300, +1100, +1700 and +1990 was found to be the best approach, conversant with the principle of economy of degrees of freedom and having regard to the smoothness of the record after +1600. The positioning of the knots is not critical, but their frequency is. Too many knots permits the curve to fluctuate unjustifiably. The resulting cubic

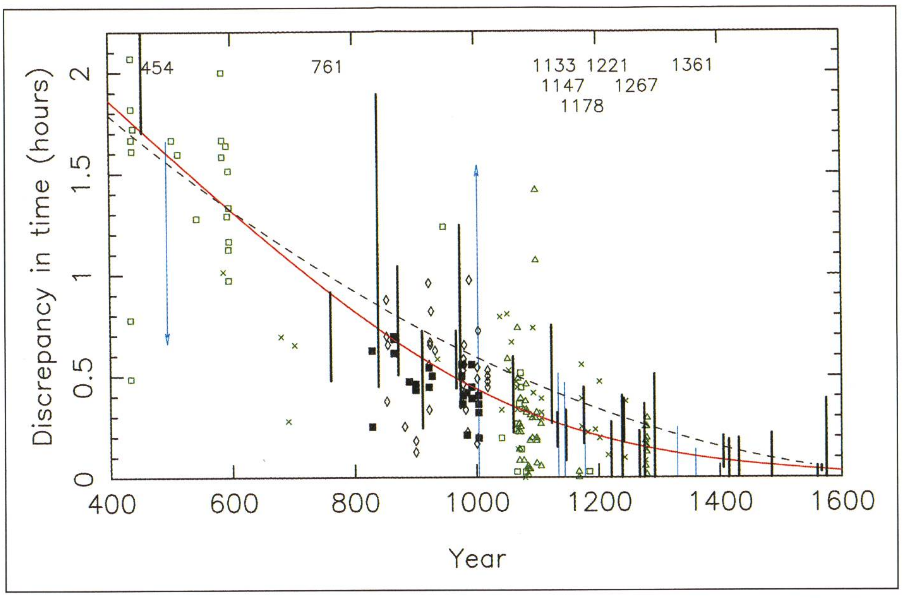


Figure 5. Plot of the critical untimed and all the timed data in the period +400 to +1605. The symbols and curves have the same significance as in Figures 3 and 4. The dates of the untimed eclipses which are in conflict with the parabola are shown at the top.

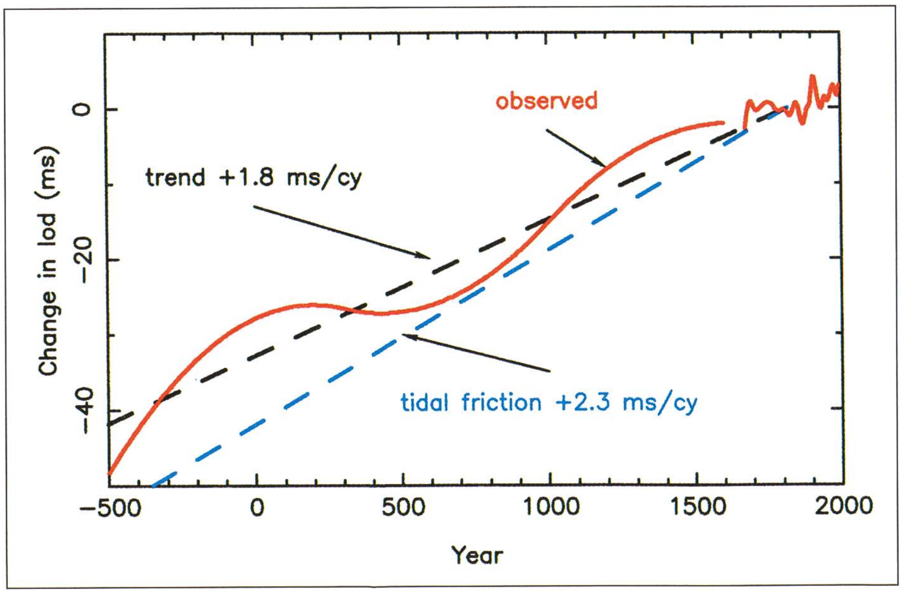
spline curve is plotted in Figures 3-5 as a continuous red curve. It satisfies all the constraints in Figure 3 imposed by the limits of the untimed solar eclipses, including all those of the 54 (less reliable) lunar eclipses, except for –382, –239, +923 and +1067.

Changes in the length of the day

By taking the first derivative along the cubic spline curve in Figures 3 and 4 we derive the change in the l.o.d. This is plotted in Figure 6, together with the fluctuations in the l.o.d. from AD 1700 to the present taken from the occultation observations which are discussed in STEPHENSON & MORRISON (1984). The first derivative of the parabola +32 t^2 produces the linear trend of +1.8±0.1 ms/cy in the l.o.d. which is equivalent to an acceleration of (–4.8±0.2) x 10^{-22} rad/s^2 in the Earth's rotation.

The observed trend of +1.8 ms/cy is obviously at variance with the predicted trend of +2.3 ms/cy due to tidal friction alone. Clearly, there is another component acting in the opposite sense which decreases the l.o.d. by –0.5±0.1 ms/cy. This non-tidal acceleration may be associated with the rate of change in the Earth's oblateness attributed to viscous rebound of the solid Earth from the decrease in load on the polar caps following the last deglaciation (PELTIER & WU,

Figure 6. Plot of the changes in the length of day (l.o.d.) –500 to +1996 obtained by taking the first time-derivative along the red curve shown in Figures 3-4. The high-frequency changes in the l.o.d. after AD 1700 are derived from the lunar occultation data.



1983; PIRAZZOLI, 1991). From an analysis of the acceleration of the node of the orbit of near-Earth satellites (CHENG et al., 1989), a present-day fractional rate of change of the Earth's second zonal harmonic J_2 of (–2.5±0.3) x 10^{-11}/yr has been derived, which implies an acceleration in the Earth's rotation equivalent to a rate of change in the l.o.d. of –0.44 ± 0.05 ms/cy. This is consistent to within the errors of measurement with our result from eclipses of –0.5 ± 0.1 ms/cy, assuming an exponential rate of decay of J_2 with a relaxation time of not less than 4000 yr.

Decade fluctuations revealed after the introduction of the telescope are no doubt present on a similar scale throughout the entire period of this investigation, but the integral of these fluctuations is too small to be detected in the pre-telescopic results. All that can be resolved is the long-term envelope of these fluctuations (see Figure 6) which, in common with the decade fluctuations, probably have their origin in core-mantle coupling (LAMBECK, 1980). The temporal behaviour of this long-term envelope is dependent on the degree of fluctuation permitted in fitting the cubic splines to the data in Figures 3 and 4. We are convinced that the fluctuation is real. Indeed, if anything, the amplitude may be greater than that shown in Figure 6, when one notes that the very coherent Arab results from solar eclipses (black squares in Figure 5) possibly indicate a greater departure from the parabola than that of our red curve. However, we note that the constraints of the lower boundaries of +1133 and +1241 do not permit a significant revision downwards. The smoothness of the observed curve after +1600 in Figure 4 also constrains the permissable degree of fluctuation on a centennial time-scale.

Conclusion

The coincidence in size of the apparent diameters of the Moon and Sun produces a startling phenomenon at the surface of the Earth which has held Man in awe. In several civilizations he has recorded these events on clay, parchment or paper, and some have survived to the present time. These records are the only way known to us at present of measuring the actual changes – as distinct from that due to tidal friction – in the Earth's rotation over the course of recorded history.

The results that we have obtained from two independent datasets for the non-tidal component of the Earth's rotation serve as a constraint on contemporary geophysical models, such as that of post-glacial uplift. More records of eclipses are probably waiting to be unearthed, and these might help fill in the gaps in Figures 3 and 4.

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Biographical information

Dr LESLIE V. MORRISON obtained his DSc from Aberdeen University, Scotland. Jointly with Professor STEPHENSON, he was awarded the Tompion Gold Medal of the Worshipful Company of Clockmakers, London for studies on Earth's past rotation. He has recently retired from the Royal Greenwich Observatory where he carried out research into Earth's rotation.

Professor F. RICHARD STEPHENSON obtained his PhD and DSc from the University of Newcastle upon Tyne, England. He is currently Professorial Fellow in the Department of Physics, University of Durham. His main research interest is in Applied Historical Astronomy. He holds the Jackson-Gwilt medal of the Royal Astronomical Society and – jointly with Dr MORRISON – the Tompion Gold Medal of the Worshipful Company of Clockmakers, London. Together with Dr DAVID A. GREEN of the Universoty of Cambridge, he is currently writing a book on «Historical supernovae and their remnants» for Oxford University Press.

L.V. MORRISON, F.R. STEPHENSON
Department of Physics, University of Durham,
GB-Durham, DH1 3LE

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