# Geometrie

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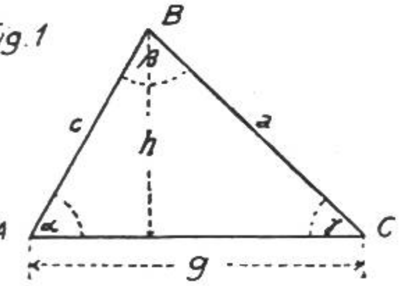
# Geometrie.

## Formeln zur Inhaltsberechnung von Flächen und Körpern.

Fig. 1




### Dreieck.

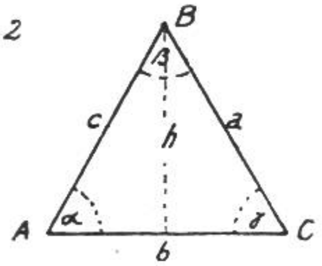
Grundlinie = $g$; Höhe = $h$; Fläche = $F$

$$F = \frac{g \times h}{2} = \frac{g}{2} \cdot h = \frac{h}{2} \cdot g;$$

$$g = \frac{2F}{h}; \quad h = \frac{2F}{g}$$

$$\sphericalangle\alpha + \sphericalangle\beta + \sphericalangle\gamma = 180° = 2R.$$

Fig. 2




### Gleichseitiges Dreieck.

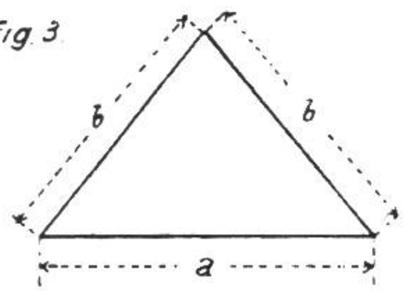
Seiten = $a = b = c$, $\angle\alpha = \beta = \gamma = 60°$

$$F = \frac{a^2}{4}\sqrt{3} = 0{,}433\,a^2$$

(genauer $0{,}4330127\,a^2$)

$$h = \sqrt{a^2 - \frac{b^2}{4}} = \sqrt{a^2 - \frac{c^2}{4}};$$

Fig. 3

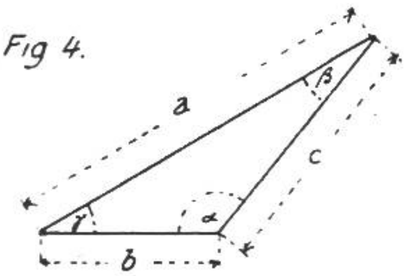



### Gleichschenkliges Dreieck.

Grundlinie = $a$; gleiche Seiten = $b$

$$F = \frac{a}{4}\sqrt{(2b+a)\cdot(2b-a)} = \frac{a}{2}\sqrt{\left(b+\frac{a}{2}\right)\left(b-\frac{a}{2}\right)}$$

Fig. 4.

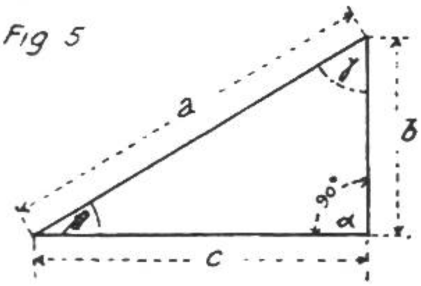



### Ungleichseitiges Dreieck.

Seiten $a$, $b$ und $c$, $s = \frac{a+b+c}{2}$;

$$F = \sqrt{s(s-a)(s-b)(s-c)}$$

Fig. 5



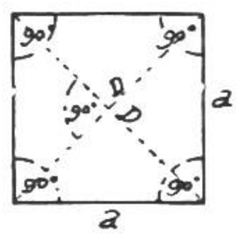

### Rechtwinkliges Dreieck. $\sphericalangle\alpha = 90°$

Hypotenuse = $a$; Katheten = $b$ und $c$,

$$F = \frac{b \cdot c}{2}; \quad a^2 = b^2 + c^2; \quad a = \sqrt{b^2 + c^2};$$

$$b = \sqrt{a^2 - c^2}; \quad c = \sqrt{a^2 - b^2}$$

Fig. 6




### Quadrat

Seite = $a$; Diagonale = $D$;

$$F = a \times a = a^2 \quad a = \sqrt{a^2}$$

$$D = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2} = a \cdot 1{,}4142.$$

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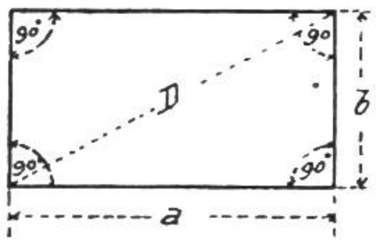
Fig. 7



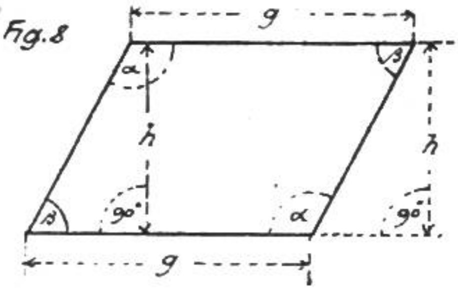

## Rechteck.

Seiten $a$ und $b$, Diagonale $D$;

$$F = a\,b;\quad a = \frac{F}{b};\quad b = \frac{F}{a};$$

$$D = \sqrt{a^2 + b^2}.$$

Fig. 8

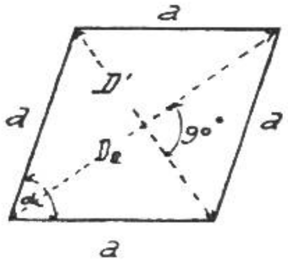



## Parallelogramm.

Grundlinie = $g$. Höhe (rechtwinklig auf Grundlinie) = $h$

$$F = g \cdot h;\quad g = \frac{F}{h},\quad h = \frac{F}{g};$$

Fig. 9

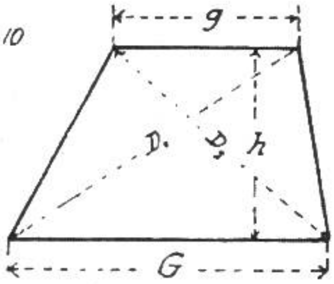



## Rhombus.

Gleiche Seiten $a$; Diagonalen $D_1$ u. $D_2$

$$F = a^2 \cdot \sin\alpha;\quad F = \frac{D_1 \cdot D_2}{2};$$

Fig 10

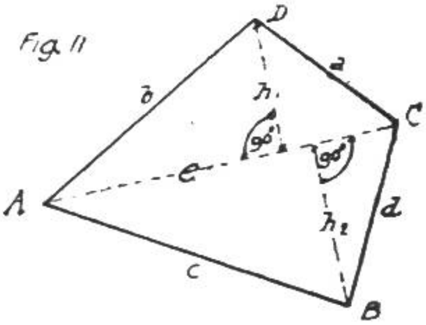



## Trapez.

Parallelseiten = $G$ und $g$, Höhe = $h$
Diagonalen = $D_1$ und $D_2$

$$F = \frac{G+g}{2} \cdot h;$$

Fig. 11

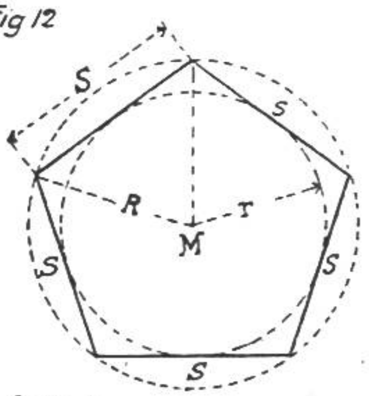



## Trapezoid.

**Diagonale $\overline{AC}$** und rechtwinklig darauf die Höhen $h_1$ und $h_2$

$$F = \frac{\overline{AC}}{2} \cdot (h_1 + h_2) = \frac{e}{2} \cdot (h_1 + h_2);$$

Fig 12




Seite = S
Radius des umschriebenen Kreises = R;
Radius des eingeschriebenen Kreises = r.

## Reguläre Vielecke (Polygon)

| Polygon | R | r | S | F |
|---|---|---|---|---|
| Dreieck | 0.577 S | 0.289 S | 1.732 R od 3.463 r | 0.433 $S^2$ od. 1.299 $R^2$ |
| Quadrat | 0.707 S | 0.500 S | 1.414 R " 2.000 r | 1.000 $S^2$ " 2.000 $R^2$ |
| Fünfeck | 0.851 S | 0.695 S | 1.176 R " 1.453 r | 1.721 $S^2$ " 2.378 $R^2$ |
| Sechseck | 1.000 S | 0.866 S | 1.000 R " 1.155 r | 2.598 $S^2$ " 2.598 $R^2$ |
| Siebeneck | 1.152 S | 1.038 S | 0.868 R " 0.963 r | 3.364 $S^2$ " 2.736 $R^2$ |
| Achteck | 1.307 S | 1.208 S | 0.765 R " 0.828 r | 4.828 $S^2$ " 2.828 $R^2$ |
| Neuneck | 1.462 S | 1.374 S | 0.684 R " 0.728 r | 6.182 $S^2$ " 2.892 $R^2$ |
| Zehneck | 1.618 S | 1.540 S | 0.618 R " 0.649 r | 7.694 $S^2$ " 2.939 $R^2$ |
| Elfeck | 1.775 S | 1.704 S | 0.563 R " 0.587 r | 9.366 $S^2$ " 2.973 $R^2$ |
| Zwölfeck | 1.932 S | 1.866 S | 0.518 R " 0.536 r | 11.190 $S^2$ " 3.000 $R^2$ |

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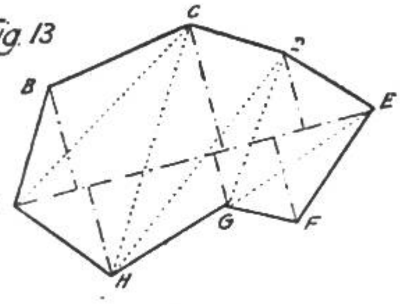
Fig. 13

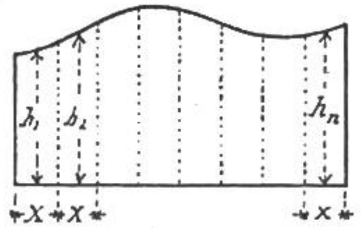



## Unregelmässige Vielecke od Flächen:

Fläche kann bestimmt werden durch Zerlegung des Vielecks in Dreiecke mittelst Diagonalen und Summirung der ermittelten Dreiecksflächen, oder durch Einteilung in Trapeze u. Dreiecke vermittelst der Koordinaten der Eckpunkte auf eine rechtwinklig gewählte Axe. Fig 13

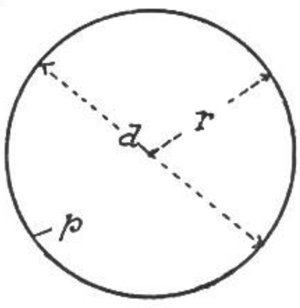
Durch Zerlegung in parallele Streifen gleicher Breite Fig 14

Fig. 14




$$F = x.(h_1 + h_2 + \ldots\ldots h_n)$$

Fig. 15




## Kreis:

Durchmesser = $d$; Radius = $r$

Umfang = $p$; Jnhalt = $F$.

$$p = d.\pi = d.3.14159$$

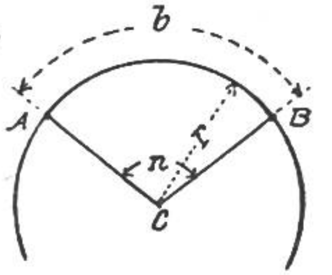
$$= 2r\pi;$$

$$F = \frac{d^2\pi}{4} = 0.785\,d^2 = r^2\pi.$$

$$r = \sqrt{\frac{F}{\pi}} = 0.564\sqrt{F};$$

$$d = 2\sqrt{\frac{F}{\pi}} = 1.128\sqrt{F}.$$

Fig 16




## Kreissektor: (A.B.C) Fig. 16.

Radius = $r$; Bogen = $b$;

Zentriwinkel = $n$;

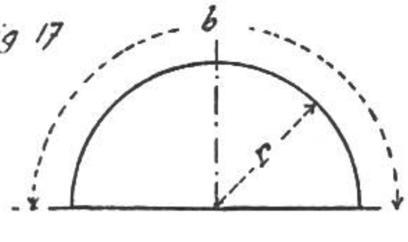
$$F = \frac{r^2\pi.n}{360} = \frac{b.r}{2}.$$

$$r\ \sqrt{\frac{F}{\pi}.\frac{360}{n}} = \frac{b}{\pi}.\frac{180}{n};$$

$$n = 360.\frac{F}{r^2\pi} = \frac{b}{r\pi}.180$$

$$b = 2r\pi\frac{n}{360} = r\pi.\frac{n}{180}.$$

Fig 17



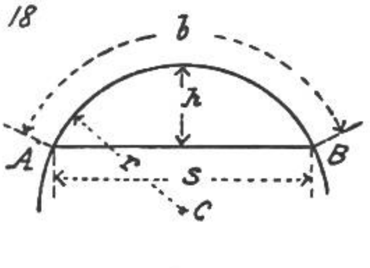

Halbkreisbogen = $b = \pi.r.$

Halbkreisfläche = $F = \frac{\pi.r^2}{2}.$

Viertelkreisbogen = $b = \frac{\pi.r}{2}$

Viertelkreisfläche = $F = \frac{\pi.r^2}{4}.$

Fig 18




## Kreisabschnitt:

Sehne = $s$. Höhe = $h$. $F = \frac{2}{3}\,s.h.$

genau. $F = \frac{r^2\pi n}{360} - \frac{1}{2}s\sqrt{r^2 - \frac{1}{4}s^2} = \frac{br - s(r-h)}{2}$

$$s = 2\sqrt{h(2r-h)} \qquad r = \frac{s^2}{8h} - \frac{h}{2}.$$

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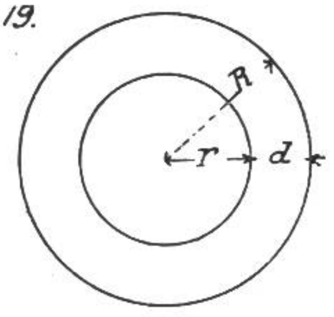
Fig 19.



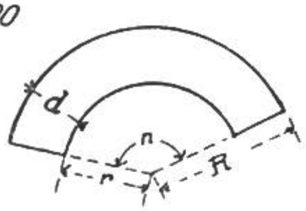

## Kreisring:

Äusserer Radius = $R$;
Innerer Radius = $r$

$$F = R^2\pi - r^2\pi.$$
$$= \pi(R+r)(R-r).$$

wenn $d$ = radiale Breite des Kreisrings, so ist $F = \pi(2r+d)d$.

Fig. 20

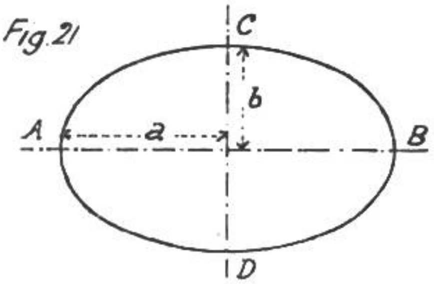



## Kreisringstück: (Konzentrisch)

Äusserer Radius = $R$
Innerer Radius = $r$
Zentriwinkel = $n$.

$$F = (R^2\pi - r^2\pi)\frac{n}{360} = (R^2 - r^2)\cdot\frac{\pi\cdot n}{360}$$
$$= (R+r)d\,\frac{\pi\cdot n}{360} = (R+r)d\cdot n\cdot 0.0087.$$

Fig. 21

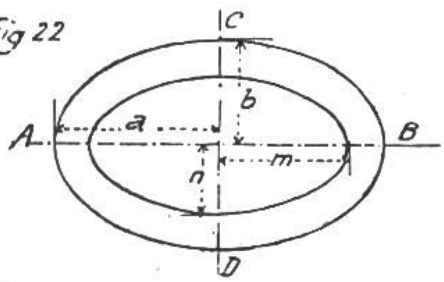



## Ellipse:

Halbe Achsen der Ellipse = $a$ und $b$

$$F = a\cdot b\cdot\pi;$$

Fig 22

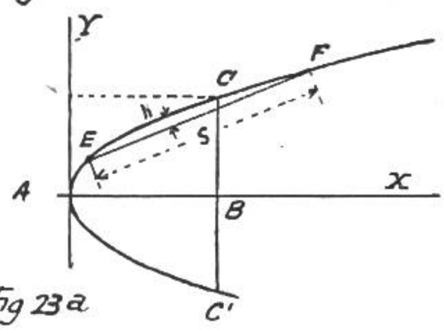



## Elliptischer Ring:

Halbe Achsen der äussern Ellipse $a$, $b$,
Halbe Achsen der innern Ellipse $m$, $n$

$$F = \pi(ab - mn).$$

Fig 23.



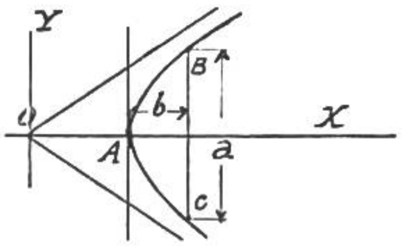

## Parabelsegment ECF:

$$F = \frac{2}{3}\,s\cdot h, \qquad s = EF.$$

## Parabelfläche CAC':

$$F = \frac{2}{3}\,CC'\cdot AB.$$

Fig 23a

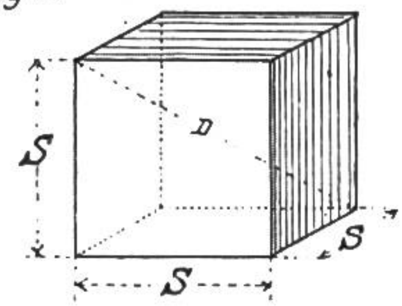



## Hyperbelsegment ABC

Sehne = $a$, Höhe = $b$

$$F\ (\text{annähernd}) = \frac{3}{5}\,b\cdot a.$$

Fig 24



## Würfel:

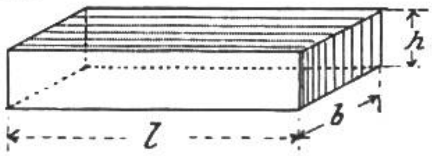
Seite = $s$; Inhalt = $K$; Oberfläche = $O$;

$$K = s^3; \quad O = 6s^2;$$

$$s = \sqrt[3]{K};$$

$$\text{Diagonale} = D = \sqrt{3s^2} = s\sqrt{3} = \\ = s \cdot 1.732050$$

Fig 25

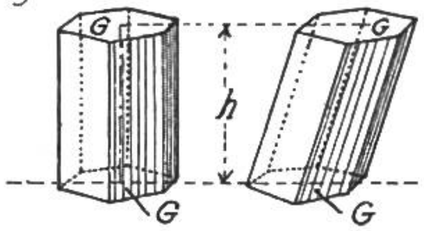


## Parallelflach:

Länge = $l$, Breite = $b$, Höhe = $h$:

$$K = l \cdot b \cdot h.$$

Fig. 26



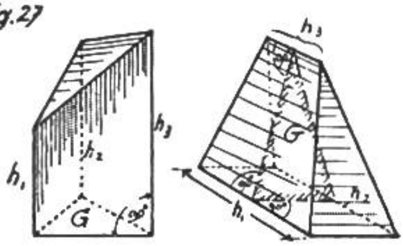
## Prisma:

Grundfläche = $G$; Höhe = $h$;

$$K = G \cdot h;$$

Oberfläche $O$ = Umfang der Grundfläche $U \times h + 2G$.

Fig. 27



## Schiefabgeschnittenes Prisma:

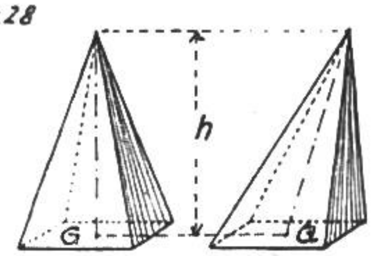
Flächeninhalt des senkrechten Querschnittes = $G$.

Länge der Kanten = $h, h_2, h_3, \ldots h_n$

$$K = G \times \frac{h_1 + h_2 + h_3 + \ldots h_n}{n}$$

Bei mehr als 3 Kanten ist das Prisma regelmässig oder hat dasselbe wenigstens Axensymetrie.

Fig. 28



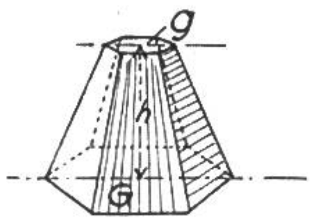
## Pyramide:

Grundfläche = $G$; Höhe = $h$;

$$K = \frac{G \cdot h}{3}; \quad h = \frac{3K}{G}.$$

$$G = \frac{3K}{h};$$

Fig 29

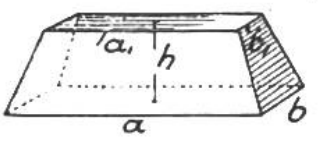


## Abgestumpfte Pyramide:

Jnhalt der beiden Grundflächen = $G$ u. $g$.

$$K = \frac{h}{3} \cdot \left(G + g + \sqrt{G \cdot g}\right)$$

Fig. 30



## Obelisk, Wall, regelmässig aufgeschütteter Haufen.

$$K = \frac{1}{6} h \left[(2a + a_1) b + (2a_1 + a) b_1\right]$$

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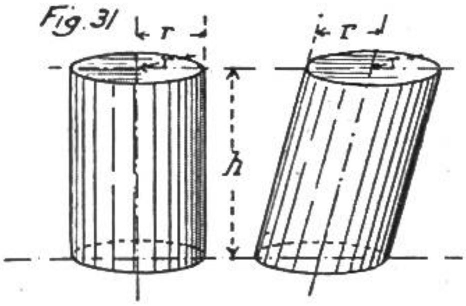
Fig. 31



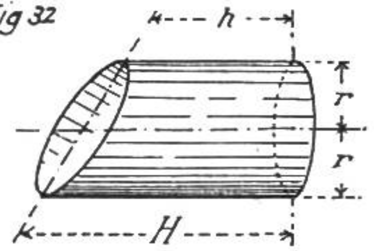
## Cylinder. (Walze)

$K = r^2\pi . h \qquad \text{Mantel} = 2 r \pi . h$

$r = \sqrt{\dfrac{K}{\pi . h}}\,; \qquad h = \dfrac{K}{r^2 \pi}\,;$

$\text{Mantel} = 2 r \pi . h .$

Fig 32



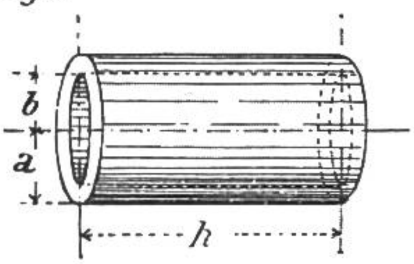
## Schiefgeschnittener Cylinder

Grösste Höhe = $H$; Kleinste Höhe = $h$;

$K = r^2 \pi . \dfrac{H + h}{2}$

$\text{Mantel} = \pi r (H + h)$

Fig 33



## Hohlcylinder:

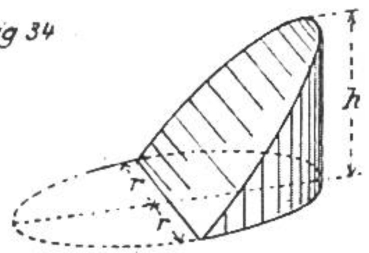
Innerer Halbmesser = $b$

äusserer Halbmesser = $a$, Länge = $h$,

$K = \pi . h (a + b) . (a - b) =$

$K = \pi . h (a^2 - b^2)$

Fig 34



## Cylinderhuf:

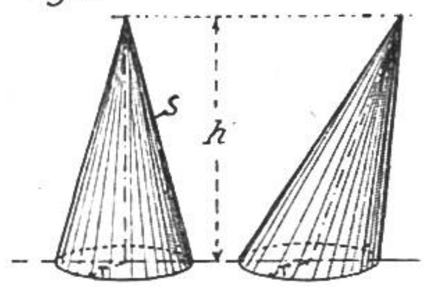
Radius der Grundfläche = $r$

Höhe des Hufes = $h$

$K = \frac{2}{3} r^2 h$

$M = 2 r . h .$

Fig 35



## Kegel.

Halbmesser der Grundfläche = $r$

Höhe = $h$, Seite = $S$

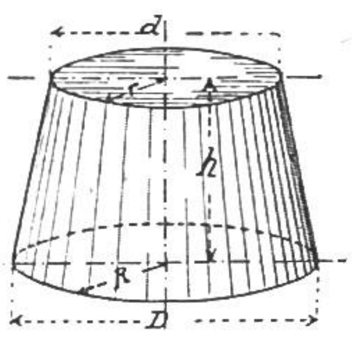
$S = \sqrt{r^2 + h^2}$

Mantel $M = r \pi . S$

Ganze Oberfläche $O = \pi . r (r + \sqrt{r^2 + h^2}) =$

$= \pi r (S + r) .$

Fig 36



$K = \frac{1}{3} r^2 \pi . h$

$r = \sqrt{\dfrac{3 K}{h \pi}}\,, \qquad h = \dfrac{3 K}{r^2 \pi}$

## Abgestumpfter Kegel

Halbmesser der Grundflächen $R$ und $r$

Durchmesser der Grundflächen = $D$ und $d$

$K = (R^2 + r^2 + Rr) \dfrac{h . \pi}{3} = \dfrac{D^2 + d^2 + d D}{12} . h \pi .$

$\text{Mantel} = \pi S (R + r)$

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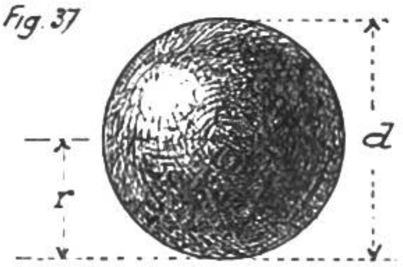
Fig. 37




## Kugel:

Radius = $r$; Durchmesser = $d$

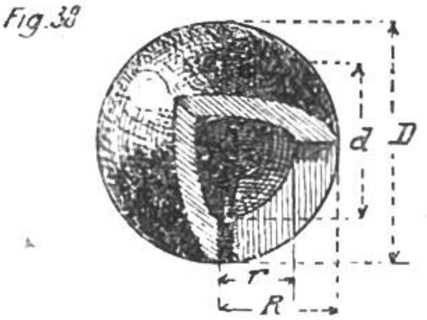
Oberfläche $O = 4r^2\pi = d^2\pi = 12.566\,r^2$

Jnhalt $K = \frac{4}{3}r^3\pi = \frac{d^3\pi}{6} = \frac{O \cdot r}{3}$

$K = 4.189\,r^3 = 0.5236\,d^3$

Radius $r = \frac{1}{2}\sqrt{\frac{O}{\pi}}; = \sqrt[3]{\frac{3K}{4\pi}}$

Fig. 38



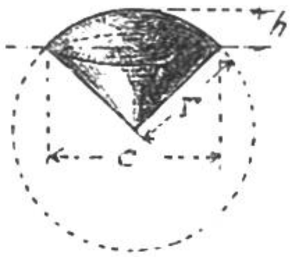

## Hohlkugel:

Äusserer Radius = $R$ innerer = $r$.

" Durchmesser = $D$ " = $d$

$K = \frac{4\pi}{3}(R^3 - r^3) = \frac{\pi}{6}(D^3 - d^3)$.

Fig. 39




## Kugelsektor:

Radius der Kugel = $r$

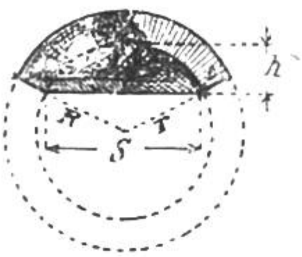
Begrenzende Kalotte: Höhe = $h$

" " Durchm = $c$

Oberfläche = $\frac{\pi r}{2}(4h + c)$

$K = \frac{2}{3}r^2\pi \cdot h = 2.0944\,r^2h$.

Fig. 40




## Hohlkugelsektor:

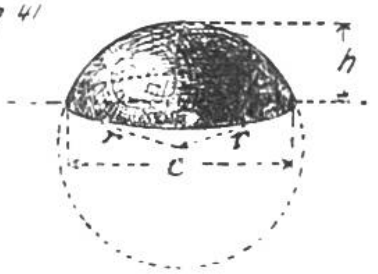
Äusserer Radius = $R$. innerer = $r$

Wanddicke = $S$; $R = r + S$.

$r = \frac{S^2 + 4h^2}{8h}$

Jnhalt $K = 2.094\,\frac{h}{r}(R^3 - r^3)$.

Fig. 41




## Kugelsegment:

Radius der Kugel = $r$.

Kalottendurchmesser = $c$ Höhe = $h$

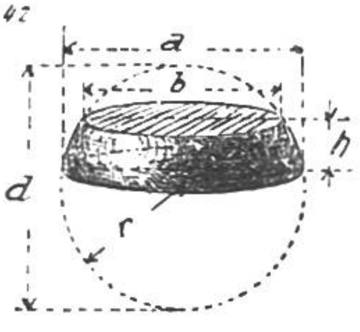
Gewölbte Oberfläche = $O$

$O = 2r\pi h = \pi\left(\frac{c^2}{4} + h^2\right)$.

$K = \frac{1}{6}\pi h(3c^2 + h^2) = \frac{1}{3}\pi h^2(3r - h)$.

$= \pi h^2\left(r - \frac{1}{3}h\right) = \pi h\left(\frac{c^2}{8} + \frac{h^2}{6}\right)$.

Fig. 42




## Kugelzone:

Höhe der Zone = $h$

Halbmesser der Kugel = $r$

Halbmesser der Endflächen $a$ und $b$

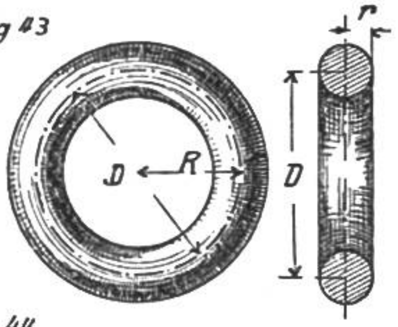
Mantel = $2r\pi h$.

Oberfläche = $M + a^2\pi + b^2\pi$.

Jnhalt $K = \frac{1}{6}\pi h(3a^2 + 3b^2 + h^2)$.

E. Pochon

Fig 43



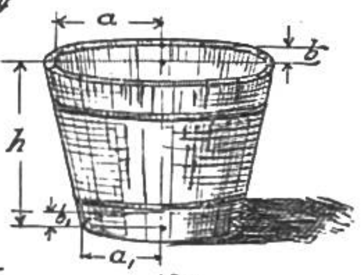

## Cylindrischer Ring.

Radius des kreisförm. Querschnittes = $r$
Durchmesser des Ringes = $D$
Radius des Ringes = $R$. siehe Figur.

$K = 2\pi^2 R r^2 = 2.467\ D d^2$.

Oberfläche $O = 4\pi^2 R r = 9.87\ D d$.

Fig. 44

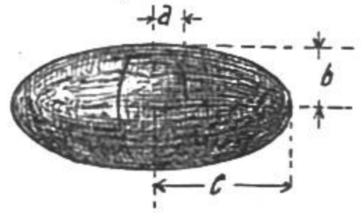



## Kübel

Die Endflächen sind Ellipsen mit den Halbachsen $a, b$ und $a_1 b_1$. Höhe zwischen den Endflächen = $h$.

Jnhalt $K = \frac{1}{6}\pi h\left[2(ab + a_1 b_1) + a b_1 + a_1 b\right]$

Fig. 45

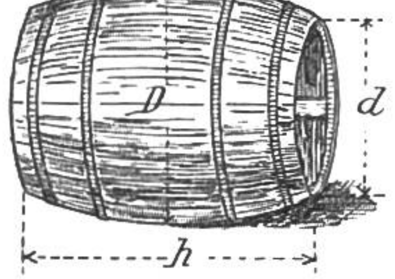



## Ellipsoid

Bezeichnung der 3 Halbachsen = $a, b, c$.

Jnhalt $K = \frac{4}{3}\pi . a . b . c$.

Fig 46



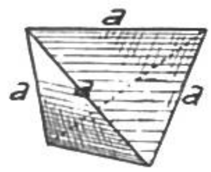

## Fass

Spunddurchmesser = $D$
Bodendurchmesser = $d$
Höhe = $h$ (resp Länge)

Jnhalt $K = 1.0453\ h\left(0.4 D^2 + 0.2 D d + 0.15 d^2\right)$

## Reguläre Polyeder:

Fig. 47

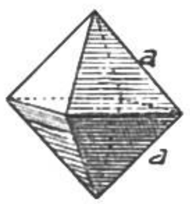



### Tetraeder: (4 gleichseitige Dreiecksflächen

Länge der Kante = $a$; $O = a^2\sqrt{3} = 1.732\ a^2$.

$K = \frac{1}{12} a^3 \sqrt{2} = 0.11785\ a^2$.

Fig. 48

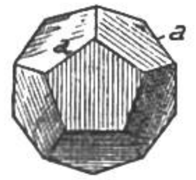



### Oktaeder: (8 gleichseit. Dreiecksflächen

Kante = $a$, $O = 2a^2\sqrt{3} = 3.464116\ a^2$.

$K = \frac{a^3}{3}\sqrt{2} = 0.4714045\ a^3$.

Fig. 49

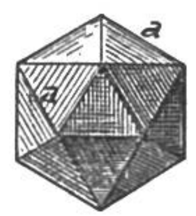



### Dodekaeder (12 regelmässige Fünfecke)

Kante = $a$; $O = 3a^2\sqrt{25 + 10\sqrt{5}}$
$= 20.645729\ a^2$.

$K = \frac{a^3}{4}\cdot(15 + 7\sqrt{5}) = 7.663119\ a^3$

Fig. 50




### Ikosaeder 20 gleichseitige Dreiecksflächen.

Kante = $a$, $O = 5a^2\sqrt{3} = 8.6602545\ a^2$

$K = 5/12\, a^3 (3 + \sqrt{5}) = 2.181695\ a^3$.