A directional constant derived from the optical indicatrix and its application in quantitative conoscopy

Autor(en): **Michaelis de Sáenz, Irene / Tessore, J.C.**

Objekttyp: **Article**

Zeitschrift: **Schweizerische mineralogische und petrographische Mitteilungen = Bulletin suisse de minéralogie et pétrographie**

Band (Jahr): **48 (1968)**

Heft 2

PDF erstellt am: **21.07.2024**

Persistenter Link: https://doi.org/10.5169/seals-37782

A Directional Constant Derived from the Optical Indicatrix and its Application in Quantitative Conoscopy

By *I. Michaelis de Sáenz* and *J. C. Tessore* (Montevideo)*)

With 8 figures and 3 tables in the text

Abstract

The change of birefringence over a definite interval of the angle of incidence, relative to the mean birefringence in this interval. is a constant that can be determined for any direction of incidence of light in the crystal. This constant k depends for a given direction on the characteristics of the optical indicatrix, and is, therefore, also related to the optic axial angle.

$$k = \frac{\Delta B}{\bar{B}} \frac{57.34}{\Delta \alpha} \qquad \begin{aligned} \bar{B} &= \text{mean birefringence,} \\ \Delta B &= \text{increment of birefringence,} \\ \alpha &= \text{angle of incidence,} \\ \Delta \alpha &= \text{increment of angle of incidence.} \end{aligned}$$

Curves of k—α were drawn from calculated data, for the symmetry planes of the ellipsoid. k can be determined directly from the conoscopic figures or better from their photographs or camera lucida drawings. Experimental k—α curves can be drawn and used for the rather accurate determination of the optic axial angle when other methods do not apply or yield only semiquantitative results. Symmetry axes can also be located as for these $k = 0$. The method is especially useful when long working-distance objectives have to be used e. g. with most heating stages.

Zusammenfassung

Die Variation der Doppelbrechung bei Änderung der Einfallsrichtung des Lichtes um einen bestimmten, nicht zu grossen Winkel steht zu diesem und der mittleren Doppelbrechung in diesem Richtungsintervall in einem konstanten Verhältnis. Es kann eine Konstante k bestimmt werden, die für eine gegebene Richtung von der Charakteristik der Indikatrix und infolge dessen auch vom optischen Achsenwinkel abhängt:

$$k = \frac{\Delta B}{\bar{B}} \frac{57.34}{\Delta \alpha} \qquad \begin{aligned} \bar{B} &= \text{mittlere Doppelbrechung,} \\ \Delta B &= \text{Inkrement der Doppelbrechung,} \\ \alpha &= \text{Einfallswinkel,} \\ \Delta \alpha &= \text{Inkrement des Einfallswinkels.} \end{aligned}$$

*) Laboratorio de Cristalofisica Aplicada, Facultad de Química, General Flores 2124, Montevideo, Uruguay.

Mit berechneten Daten wurden k—α-Kurven hergestellt. Diese können zur relativ genauen Bestimmung des optischen Achsenwinkels gebraucht werden, wenn andere Methoden nicht anwendbar sind oder nur annähernde Resultate geben. Die Lage der Symmetrieachsen der Indikatrix kann bestimmt werden, weil bei diesen $k=0$ ist. Die Anwendung der Kurven ist besonders nützlich, wenn Objektive mit grossem Arbeitsabstand benutzt werden müssen, z. B. bei den meisten Heiztischen.

INTRODUCTION

Literature on quantitative conoscopy is not very abundant. The most frequently used method is the well-known determination of the optic axial angle from the distance between the isogyres by MALLARD'S constant when both isogyres fall into the field or the position of one isogyre and one bisectrix of the optic axial angle can be located. For determinations with other crystal orientations BEREK defined the "characteristic function of birefringence" which can be used for determining $2V$ as long as one symmetry plane and one symmetry axis can be accurately located in the field (RINNE, BEREK, 1953). The method of BECKE and WRIGHT also described by these authors is based on FRESNEL'S construction and can be used when one optic axis is in the field. The optic axial angle is derived from the vibration directions located in the interference figure. This method presents serious difficulties in its application. RITTMANN (1963) discusses various applications of quantitative conoscopy. Actually his considerations are based on the same principle as BEREK'S method. Instead of relating the retardations corresponding to different inclinations in the same symmetry plane, he relates the retardations determined at the same inclination in two perpendicular symmetry planes. The results obtained by RITTMANN'S procedures are mostly semiquantitative but have the advantage of being rapid. For centered symmetrical figures the results are more accurate. BECKE'S determination of the optic axial angle from the "exit angle" of the isogyres first proposed by MICHEL-LÉVY is also discussed by RITTMANN (1963). It is a rather rough estimate—the same as the estimation of $2V$ from the curvature of the isogyres proposed by BECKE (compare BURRI 1950). MALLARD'S method was also applied by BRYANT (1941) to photographs of the interference figures. The method is most interesting in some of its applications. It consists in determining the acute and obtuse optic axial angle using inmersion lenses of high aperture. $2V$ can so be determined without the refractive index of the substance being known. It is not always possible to apply this method because of the small working distances of the objectives and the necessity of rotating the crystal.

During a study of the phase transformations in crystals under the microscope using heating stages, the necessity of employing a different method became apparent, as objectives of high angular aperture could not be used on account of their small working distances. Usually the crystals could not be moved. Furthermore time-consuming measurements with the eyepiece micrometer were often impossible as the speed of transformation of the crystals was sometimes too rapid. The changes in orientation were frequently so considerable that BEREK's method could not be applied as in one of the positions no symmetry axis was present in the field even when the UMK—50/0.60 (Leitz) objective was used. We also found that when only one retardation was determined in addition to the one at the symmetry axis, as recommended by BEREK, the results were not accurate at all.

To make possible careful angular measurements the determinations were made on the photographs of the conoscopic figures. This also allows more than one determination per minute to be made if required by the speed of transformation of the sample. In this way accurate birefringence or retardation curves can be drawn as proposed by BEREK (RINNE, BEREK, 1953) for determinations with the universal stage. Plotting $R^* = R \cos \alpha$ against α the maxima and minima locate the symmetry axes and the values of $R = 0$, the optic axes if these are in the field. This photographic method was successfully applied at temperatures above 100° C by A. LORENZI (1961) in our laboratory (personal communication).

In the method proposed in this paper it is convenient to use the photographic technique because accuracy is much higher. Exact results can also be obtained by drawing the interference figure with the camera lucida. Care must be taken that the central ray is exactly perpendicular to the plane of the drawing paper. A circle is drawn whose diameter equals that of the perimeter of the field image for a certain enlargement. Coincidence of the perimeter of the field and this circle should be checked for each determination. This method is not convenient for rather rapid transformations.

The method can be also used with less precision by using eyepiece-micrometer measurements to determine the inclination angles. The results are rather poor, however.

THE CONSTANT OF ANGULAR VARIATION OF BIREFRINGENCE

The variation of birefringence related to the mean birefringence in a determined interval of the angle of incidence is a constant for a given direction of incidence in any crystal.

The variation of the refractive indices in an anisotropic substance, with the direction of the wave normal is represented by the indicatrix and can be calculated by FRESNEL's formula:

$$n_x = \frac{n_g n_p}{\sqrt{n_g^2 \sin^2 \alpha + n_p^2 \cos^2 \alpha}},$$

where n_x is the refractive index in an inclined direction in a principal section

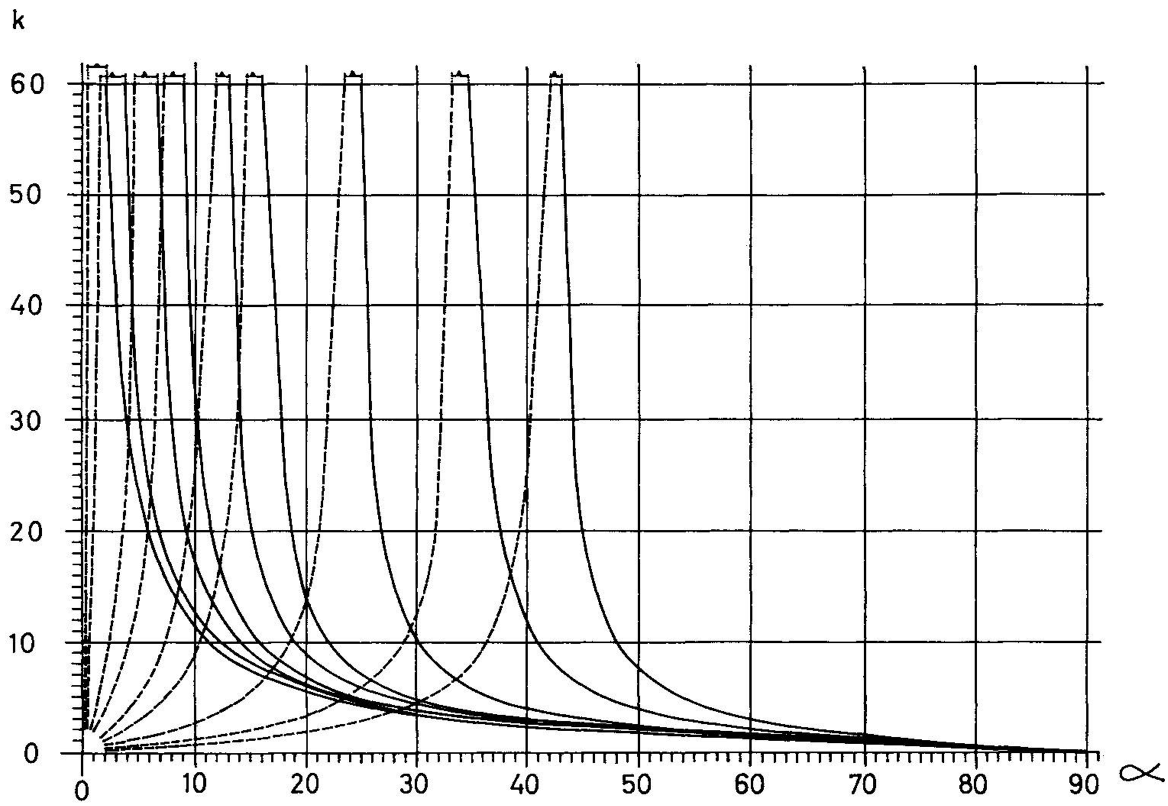

Fig. 1. Plane A.B.—O.B. The variation of k with in the optic axial plane is represented. As birefringence has a maximum value at the symmetry axes contained in this plane, the constant k is 0. It increases to ∞ at the optic axes and, as the birefringence changes in sign, k varies from $+\infty$ to $-\infty$; the absolute value then decreases from ∞ to 0 at the other symmetry axis. For convenience in the application of these graphs only the absolute values of k were plotted. Close to the symmetry axes (especially for the larger optic axial angles) the curve k—α is nearly a straight line. The slope $\operatorname{tg} \rho = \frac{10k}{\alpha}$ can be used as a characteristic value. Its relation to the optic axial angle is plotted in fig. 4. In the rectilinear portion of the curve $\operatorname{tg} \rho = k_{10}$; this is the value of k at 10° from the bisectrix. k_{10} is useful for even smaller optic axial angles in the portion where the curve is not rectilinear; the values of k for these cases are given in curve B fig. 4.

of the ellipsoid, n_g is the larger index in that plane and n_p the lower one; α is the angle of the wave normal with one of the symmetry axes of that plane. The refractive index n_k of the wave vibrating normal to the plane remains constant so that the birefringence for any direction in that plane is: $B_x = n_k - n_x$.

As the birefringence varies with the inclination this variation of n_x can be related to the angular variation of the direction of incidence and also to the mean birefringence in that interval, which will be equal to the birefringence in that direction if infinitely small intervals are taken. A coefficient C can be defined:

$$\frac{dB}{d\alpha}\frac{1}{B_0} = C.$$

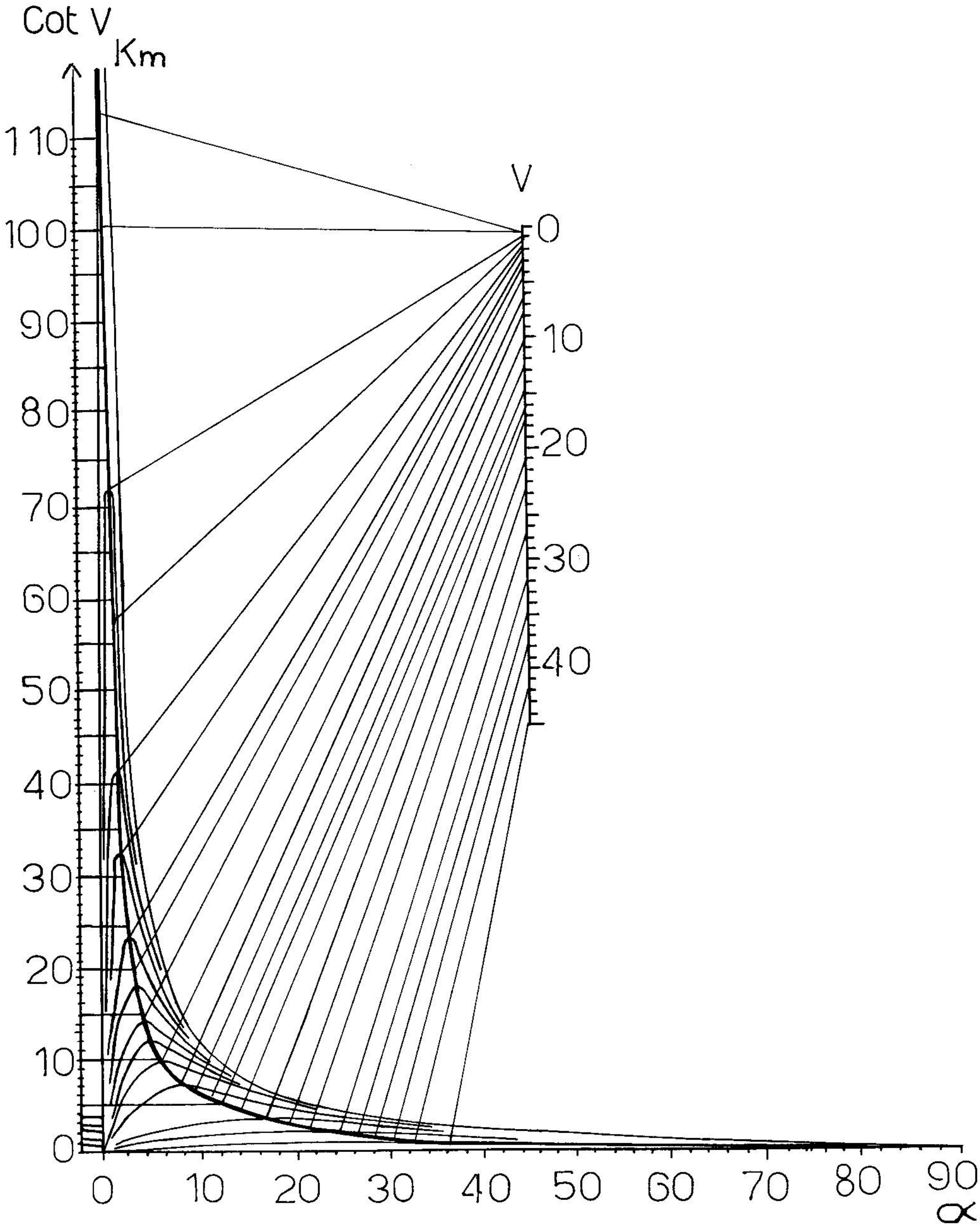

Fig. 2. Plane A.B.—O.N. It is seen that k increases from 0 at the symmetry axes to a maximum value in the A.B.—O.N. plane. In a considerable range of V-values $k_{max} = \cot V$ so that V can be determined directly from this value even without the use of the graphs. This is true for $2V$ values smaller than approx. 45°. In the straight portion near the symmetry axes $\operatorname{tg} \rho$ or k_{10} values can be also used to find $2V$ (see fig. 4).

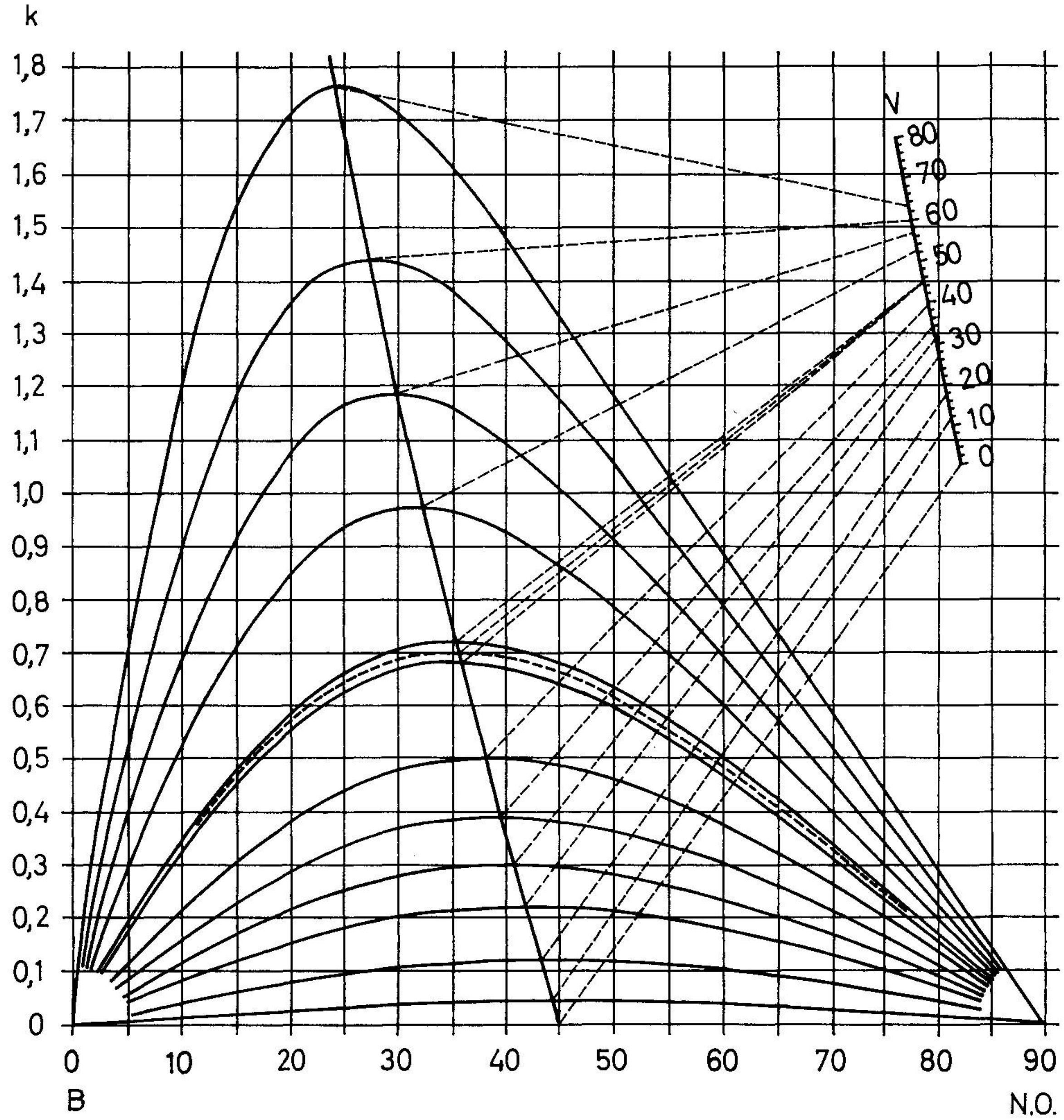

Fig. 3. Plane $B_{(O.A.)}$—O.N. This nomogram is similar to that of fig. 2 but drawn in a larger scale as for the large values of $2\,V$ and especially for the obtuse optic axial angles the k-values are very low. In this zone it is rather difficult to determine k_{max} accurately but $\operatorname{tg} \rho = k_{10}$ is very sensitive to small changes of $2\,V$ for the large optic axial angles. The value of V can be accurately determined even in rather inclined figures, as the rectilinear part extends far from the symmetry axes.

In practice small increments must be taken, so the mean coefficient is used:

$$\frac{\Delta B}{\Delta \alpha} \frac{1}{B_0} = \bar{C},$$

where B_0 is the initial birefringence (for other definitions see the abstract).

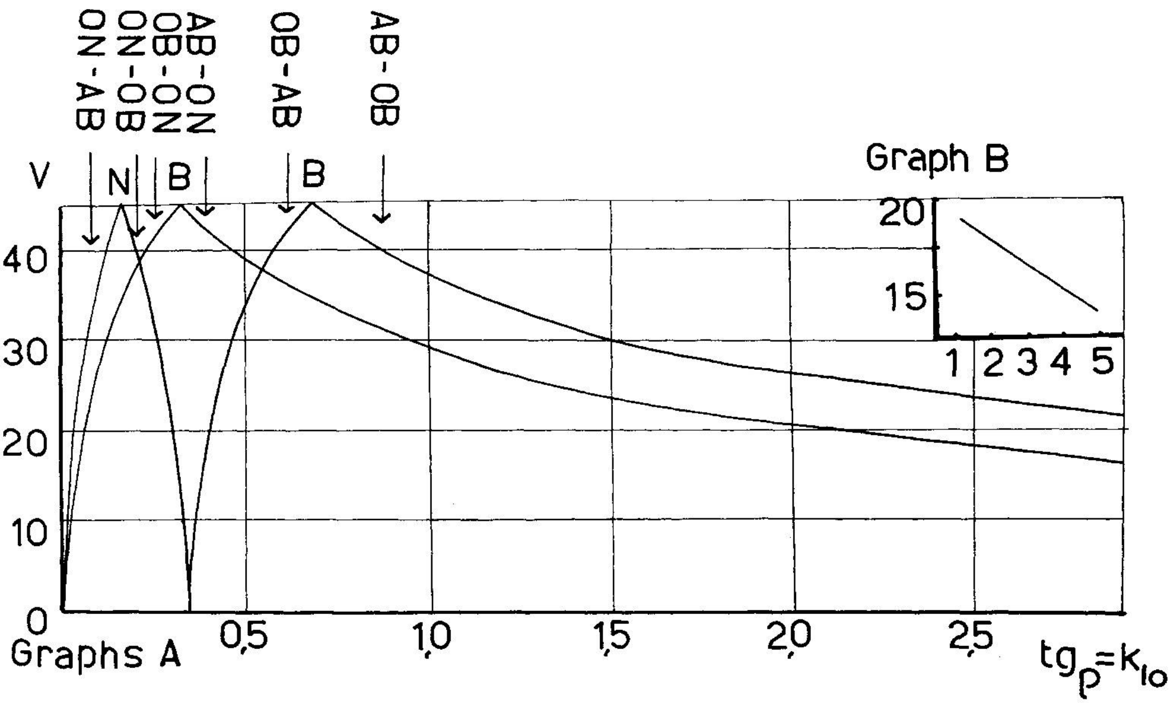

Fig. 4. Graphs A represent the slopes of the rectilinear part of the k—α curves near the symmetry axes of the ellipsoid $k_{10} = \mathrm{tg}\ \rho = \frac{10\,k}{\alpha}$. Graph B represents k_{10} in the nonrectilinear part of the curves, that is for those corresponding to the lower optic axial angles. k_{10} is no longer the slope but is nevertheless a useful value for determining $2\,V$ when the optic axes are not in the field. The curves are most useful for the A.B.—O.B. plane, even if the bisectrix is somewhat out of the field and no O.A. is seen. In no case it is necessary to have centered figures or symmetrical ones. The $\mathrm{tg}\ \rho$ values for the O.N.—$B_{(O.A.)}$ planes are not very sensitive but can be used in an emergency for an approxinate estimation of $2\,V$.

In practical application it was found to be more convenient to relate the variation of birefringence to the mean birefringence of the interval, instead of to the initial one. A constant is thus determined which is practically undependent of the variation of the thickness of the crystal with inclination. This constant is:

$$\frac{\Delta B}{\Delta \alpha} \frac{1}{\bar{B}} = \mathrm{k}.$$

As the intervals examined are usually small (5—10°) it can be admitted that $\frac{\Delta B}{\bar{B}} = \frac{\Delta R}{\bar{R}}$ where R is the retardation expressed e.g. in mμ, as determined for that direction; thickness need not to be taken into account.

As explained above the maximum value of this constant can be used for the determination of V from measurements in the $B_{(O.A.)}$—O.N. plane.[1])

[1]) A.B. = acute bisectrix; O.B. = obtuse bisectrix; $B_{(O.A.)}$ = bisectrix of O.A. angle; O.N. = optic normal; O.A. = optic axis.

It was found that for most values of V, $k_{max} = \cot V$ if $k = 57.34\, K$. Therefore the constant k is used throughout this paper:

$$k = \frac{\Delta B}{\bar{B}} \frac{57.34}{\Delta \alpha}.$$

57.34 is an empirical factor.

The variation of the constant k with the direction of incidence was calculated with the I.B.M. 1401 computer, for the symmetry planes and is represented in figures 1, 2 and 3. Fig. 4 represents the k-values at 10° from the symmetry axes of the ellipsoid (see text corresponding to the figures).

EXPERIMENTAL METHOD

In order to determine the constant $k = \frac{\Delta B}{\bar{B}} \frac{57.34}{\Delta \alpha}$ it is necessary to measure the retardation for certain known directions within a symmetry plane of the ellipsoid. The angular distance between these directions has to be determined. This can be accomplished by rotating the crystal around an axis perpendicular to that plane which should be vertical. Alternatively the retardation for vibrations with different inclinations to the microscope axis can be determined by examining the conoscopic figure. The best way of obtaining these angular values was found to be the application of MALLARD's method to photographs or camera lucida drawings of the conoscopic figure. MALLARD's constant is determined as usual from the aperture of the objective and the diameter of the field. Points of known retardation are usually located by taking the intersection of the symmetry plane with the isochromatic or rather the dark bands of the interference figure. Retardation at these points is $n\lambda$. For narrow bands it can be admitted that the light bands correspond approximately to a retardation of $(n/2)\, \lambda$; if it is necessary to locate points with $\Delta R = (1/4)\, \lambda$ the mica plate can be used and the displacement of the bands noted. If it is required to know the retardation at a particular point e.g. at the symmetry axis, this may be accomplished using BEREK's compensator. This is not always necessary but can be done to verify the results.

The angular distances between the points of known retardation are found directly using WINCHELL's chart (TRÖGER 1956) or calculated from MALLARD's constant if the symmetry plane contains the microscope axis. If the symmetry plane is inclined a gnomonic projection is drawn as follows:

The angle of the vertical rotation axis of the microscope stage with the direction of known retardation, is determined as explained above from MALLARD's constant (α). Then the photograph is fixed with its center on the center of the tangent net (fig. 5). Lines are drawn on tracing paper from the center of the photograph through the points of known retardation (fig. 6). The

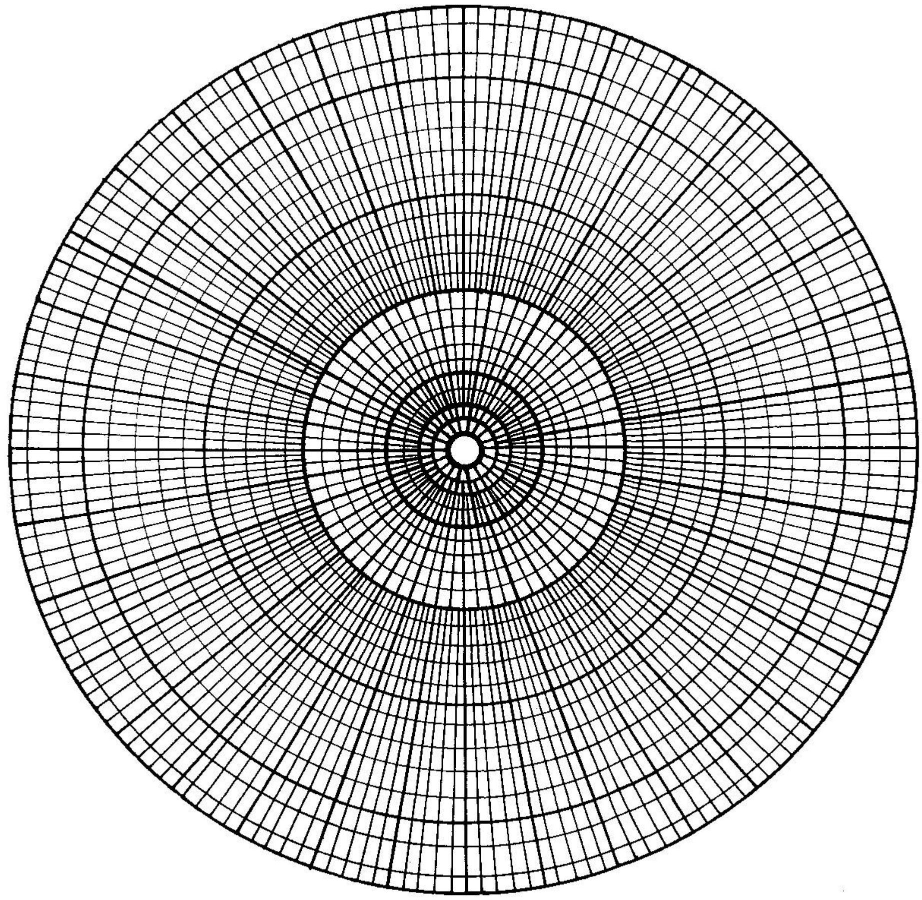

Fig. 5. Tangent net. The net is used to draw on a large scale the gnomonic projection of directions with rather small inclinations to the microscope axis. (See experimental method.) The concentric circles are drawn at intervals of two degrees and the perimeter lies at 45°. For large angles the stereographic net or PARKER's gnomo stereographic net can be used (see TERPSTRA and CODD, 1961 or the original papers).

azimuth of these points φ can so be determined directly. The photograph is taken off and on the tracing paper the values corresponding to the angle of the directions P_n with the microscope axis are plotted on each line taking into account that on the net the circles are 2° apart and the perimeter lies at 45°. A line is now drawn through all these points of known retardation (P_1, P_2, P_3 etc.) which should lie on a straight line: the gnomonic projection of the symmetry plane.

Then the angle point of the projection is found by drawing a parallel to the line P_1, P_2 etc. as well as a perpendicular through the center C (fig. 7). The distance $\overline{DM_n}$ is now measured and drawn on the perpendicular $\overline{CD}$ or its prolongation. The angle point R is so determined (compare TERPSTRA and CODD, 1961). The angle point R is now fixed in the center of the net and comparing the lines drawn from R through P_1, P_2 etc. the azimuth angle is read directly.

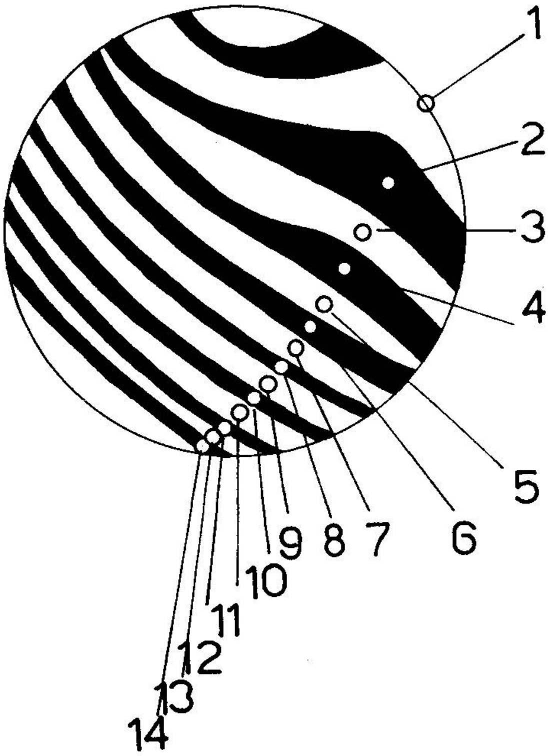

Fig. 6. Sketch of the photograph corresponding to the practical example. No symmetry axes and no O.A. are in the field. Objective UMK 50/0.60 (Leitz); $\lambda = 480$ mμ; $2V$ determined with the universal stage: $44° \pm 1$. (Compare table 2 and fig. 7.)

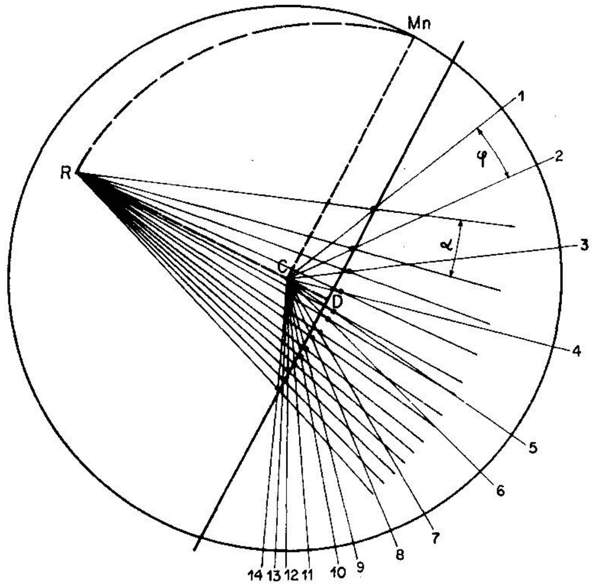

Fig. 7. Geometrical construction explained with the experimental method (p. 479). $\overline{CM_n}$ = parallel to the gnomonic projection of the symmetry plane; $\overline{CD}$ = perpendicular to this line; R = the angle point located on the prolongation of the perpendicular $\overline{CD}$; φ azimuth of the directions of known retardation; α = angle between these directions read on the tangent net fixed with R at its center.

To avoid tedious calculations regular intervals of R are usually taken, as explained above. The relation: $\frac{\Delta B \cdot 57.34}{\bar{B}}$ called M can be taken from table 1 that applies independently from the chosen wavelength and from the intervals that were taken. As $\frac{\Delta B}{\bar{B}} = \frac{\Delta R}{\bar{R}} = \frac{\Delta R}{R_1 + \Delta R/2}$ expressing $\bar{R}$ and ΔR as multiples or fractions of the given wavelength: $\Delta R = n\lambda$ and $R_1 = N\lambda$;

$$\frac{\Delta B}{\bar{B}} = \frac{n}{N + n/2}.$$

The values for M are given in table 1 for each $\bar{R}$ up to $15n$.

For example: The retardation for two points which are 5° apart are 2 and $(2\frac{1}{2})\lambda$; $\bar{R} = (2 + \frac{1}{4})\lambda$; $n = \frac{1}{2}$; $\bar{R} = (4 + \frac{1}{2})n$; $\Delta R = (\frac{1}{2})\lambda$; M taken from table 1 is 12.7 and k can be calculated with the slide rule being: $k = \frac{M}{\Delta\alpha} = 2.54$.

In the same way k-values can be rapidly calculated for the other directions of the symmetry plane and the curve for k—α can be drawn. Comparing the

Table 1

$\bar{R}$	M	$\bar{R}$	M	$\bar{R}$	M	$\bar{R}$	M
0 n	—	4 n	14.3	8 n	7.2	12 n	4.8
(½) n	114.7	(4½) n	12.7	(8½) n	6.8	(12½) n	4.6
1 n	57.3	5 n	11.5	9 n	6.4	13 n	4.4
(1½) n	38.2	(5½) n	10.4	(9½) n	6.0	(13½) n	4.3
2 n	28.7	6 n	9.6	10 n	5.7	14 n	4.1
(2½) n	22.9	(6½) n	8.8	(10½) n	5.5	(14½) n	4.0
3 n	19.1	7 n	8.2	11 n	5.2	15 n	3.8
(3½) n	16.4	(7½) n	7.6	(11½) n	5.0		

experimental values with those given in figs. 1—4 the value of V can be found and the symmetry axes located.

PRACTICAL EXAMPLE

Although many determinations were carried out to verify and apply this method only one example shall be described in detail. The results of others are given briefly.

An inclined figure of muscovite photographed with the UMK 50/0.60 Leitz objective is reproduced in fig. 6. Points were located for every (½) λ on the symmetry plane. It was considered convenient to make the calculations for intervals of 1 λ as for (½) λ the angular increments would be too small and a considerable error would be incurred. Lines were drawn from the center of the field through points P_1, P_2 etc. as explained above (fig. 7). The distance $C\,P_n$ was carefully measured for each line and α calculated from MALLARD's formula with the slide rule (WINCHELL's diagram reproduced by TRÖGER [1956, p. 122]

Table 2. *Practical example: neither symmetry axes nor optic axes are in the field (compare fig. 7 and 8)*

Objective: UMK $50/A = 0.60$; $\lambda = 480\,\mathrm{m}\mu$; $2\,V_\alpha = 44°$; $\Delta R = 1\,\lambda = n$
(for values of M see table 1)

N°	d	α	α_{PS}	R	M	$\Delta\,\alpha$	k
1	5.0	21.5	0	2 n	—	—	—
2	3.4	14.0	9	(2½) n	22.9	14.0	1.6
3	2.7	12.0	14	3 n	19.1	9.0	2.1
4	2.4	11.0	18	(3½) n	16.4	8.0	2.1
5	2.5	11.5	22	4 n	14.3	6.0	2.4
6	2.5	11.5	24	(4½) n	12.7	5.0	2.5
7	2.8	12.5	27	5 n	11.5	5.0	2.3
8	3.1	13.5	29	(5½) n	10.4	4.5	2.3
9	3.5	15.0	31.5	6 n	9.6	4.0	2.4
10	3.8	16.5	33	(6½) n	8.8	3.5	2.5
11	4.0	17.5	35	7 n	8.5	4.0	2.1
12	4.4	18.5	37	(7½) n	7.6	4.0	1.9
13	4.6	20.5	39	8 n	7.2	4.0	1.8
14	4.8	22.0	41	(8½) n	—	—	—

can also be used). The α values are drawn on the projection using the tangent net. The P-points lie more or less on a straight line. This line is drawn and the angle point found as explained above. This point R is now fixed in the center of the tangent net and the angles between the directions corresponding to these points are read (see values in table 2).

From these values the curve k—α is now drawn (fig. 8) and it is seen that two characteristic values are obtained: $k_{max} = 2.5$ and $k_{10} = 1.4$. A comparison with fig. 2 and 4 shows that these values correspond to $2V = 44$ and 48. The real angle as determined with the universal stage was 44° which is in rather good agreement, considering that other methods do not apply under these conditions. In this example the main error consists in the determination of the

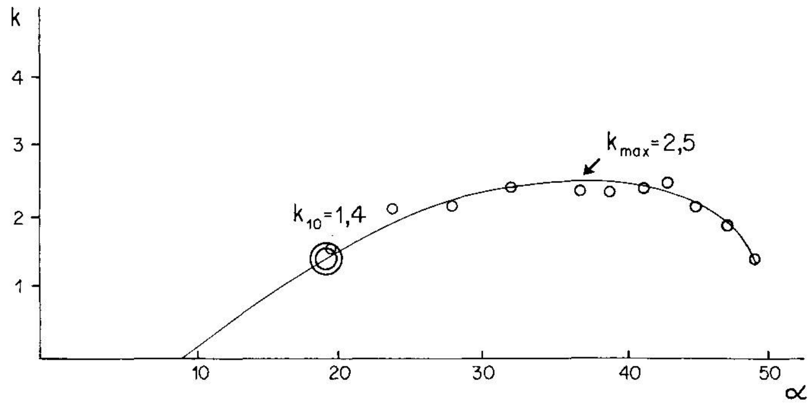

Fig. 8. k—α curve for the practical example (see fig. 6 and table 2); α = the angle of incidence referred to a certain direction of the interference figure contained in the symmetry plane. The size of the circle representing k_{10} gives an estimate of the error incurred in its determination.

$$k = \frac{\Delta B}{\bar{B}} \frac{57.34}{\Delta \alpha}; \quad k_{max} = 2.5; \quad k_{10} = 1.4; \quad 2V = 44°.$$

Table 3. *Examples of other determinations*

Orientation	k	V	V^+
1. A.B. 10° out of the field the S.P. contains the microscope axis. Plane: A.B.—O.N.	$k_{10} = 0.038$	17°	18°
2. A.B. out of the field. The S.P. contains the O.N. Plane: A.B.—O.N.	$k_{10} = 1.7 \pm 0.2$	$22.5 \pm 2.5°$	22°
3. A.B. centered. No O.A. is in the field. Plane: A.B.—O.N.	$k_{10} = 1.7 \pm 0.1$	$22.5 \pm 1°$	22°
4. Inclined asymmetric figure. No S.A. in the field. Plane: A.B.—O.N.	$k_{10} = 1.4$	$24.0 \pm 2.5°$	22°
	$k_{max} = 2.5$	$22.0 \pm 2°$	22°
5. O.B. nearly centered. Low birefringence.			
Plane: O.B.—O.N.	$k_{10} = 0.13$	$32.3 \pm 3°$	29°
Plane: O.B.—A.B.	$k_{10} = 0.50$	$33.5 \pm 3°$	29°
6. 2 V near to 90°. O.B. in the field or near.	$k_{10} = 0.20$	$40.0 \pm 3°$	43.5°

Note: V^+ is optic axial angle measured with the universal stage. S.A. = Symmetry axis; S.P. = Symmetry plane.

position of the symmetry plane. Small deviations do not affect the results to any great extent.

Results of some other determinations are given in table 3.

DISCUSSION AND APPLICATIONS

It is difficult to make a general estimation of the errors of the method, as a variety of factors that can not be quantitatively estimated affect the accuracy of the results. Some general conclusions can however be drawn.

The magnitudes that are measured are the azimuth and the angle of incidence α referred to the direction of the axis of rotation of the microscope—stage, and the retardation R. If the determination is made on an interference figure the retardation can be determined rather accurately using monochromatic interference filters. The error of ΔR is small compared with that of $\Delta \alpha$. The latter is usually rather large if the measurements are made directly with the eyepiece micrometer. This is also one of the main errors of the method proposed by Berek. In the photographs the distance from the center of the field to the point of known retardation can be determined with good accuracy. Determinations with a properly checked camera lucida are also reliable. If the interference bands are broad or diffuse accuracy diminishes. The azimuth is always found with a considerably higher accuracy with the latter methods. However, for small angular increments the relative error is likely to be quite large. From our experience intervals between 5 and 10° are the most convenient ones. When large increments are taken the accuracy is apparently increased. This is true in the straight portion of the k—α curves, but the chance of missing the maxima of the curves or skipping the part tending to ∞ is greater, and the uncertainty of a curve drawn with very few points also arises.

The first advantage of this method is that conoscopic figures obtained with long working-distance objectives can be quantitatively interpreted. Limits to orientation conditions are very few compared with the usual methods. For example it is seen that large optic axial angles can be accurately determined either on inclined A.B. or O.B. figures with no O.A. in the field. In fact in these rather straight curves $\operatorname{tg}\rho = k_{10}$ can be found with an error of about 10%. For $2V = 80$ for the O.B. figure, k_{10} is 0.22 and for $2V = 90$ it is 0.32, for $2V = 100$ (or—80) it is 0.48. A considerable precision can be obtained as long as the figures show clear interference bands. The values given here correspond to the $B_{(O.A.)}$—O.N. plane (see fig. 4). In the optic axial plane similar results are obtained. With objectives of large angular aperture nearly every possible position of the crystal can be used for the determination as it is very rare that no symmetry plane falls into the field under these circumstances. This allows the method to be extended to flaky crystals or crystal needles of which it may be difficult to make oriented slides. In rock slides also orientations convenient for the application of the other methods are often not to be found; the method may then be useful if it is desired not to use the universal stage.

We have seen that the greatest error in the determination of k arises from the determination of the increment of the angle of incidence from the interference figure. It is possible to apply the method with stages with at least one horizontal axis. (If a 5-axis stage is available it is certainly better to apply BEREK's method based on the characteristic function of extinction.) With spindle stages or two-axis stages the method can be useful and can give very accurate results. Spindle stages may also be adapted to heating experiments.

With a stage with a horizontal axis the error in the determination of α is very small. The greatest trouble now arises in the accurate determination of R if a high precision is desired. Using MOSEBACH's method (1949) with the BEREK compensator the precision of the determination of R is considerably increased. Small angular intervals can now be taken as long as the relative error in ΔR does not increase too much. In our laboratory special compensators for the exact determination of low retardations are not available. As birefringence is always low near the optic axes and therefore also near the acute bisectrix of crystals with small $2V$, the maxima of these crystals could not be measured. However, when good measurements of R can be attained with special compensators a very high precision in the determination of small optic axial angles should be obtained, as $k_{max} = \cot V$. Angles smaller then $2V = 10°$ would be specially sensitive (compare fig. 2).

Acknowledgements. We wish to thank Dr. R. Carbó of the Chemical Institute of Barcelona for determining the program for the calculations with the IBM 1401 computer, A. Lorenzi for trying out the photographic technique, and M. Nalerio for several drawings.

REFERENCES

BEREK, M. (1953): Rinne-Berek. Anleitung zur optischen Untersuchung mit dem Polarisationsmikroskop. Herausgegeben von Ch. Claussen, A. Driesen und S. Rösch. Schweizerbart'sche Verlagsbuchhandlung, Stuttgart.

BRYANT, W. M. D. (1941): Optical crystallographic studies with the polarizing microscope III. Measurement of several types of selective dispersion in organic compounds. J. Amer. Chem. Soc. *63*, 511—516.

BURRI, C. (1950): Das Polarisationsmikroskop. Verl. Birkhäuser, Basel.

LORENZI, A. (1961): Estudio óptico sobre el polimorfismo del KNO_3. Trabajo especial. Facultad de Química. Montevideo. (Unpublished.)

MOSEBACH, R. (1949): Heidelb. Beitr. zur Min. und Petr. *2*, 172—175.

RITTMANN, A. (1963): Quantitative Konoskopie. Schweiz. Min. Petr. Mitt. *43*, 11—36.

TRÖGER, W. E. (1956): Optische Bestimmung der gesteinsbildenden Minerale. Teil I. Schweizerbart'sche Verlagsbuchhandlung, Stuttgart.

TERPSTRA, P. and CODD, L. W. (1961): Crystallometry. Longman's Green and Co., London.

Manuscript received May 30th, 1967.